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Bayesian updating of the residual structural performance of existing concrete structures based on experimental tests

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ABSTRACT

Reliability analysis of existing concrete structures requires performance estimations which are affected by uncertainties related to system geometry, material characteristics, loading scenarios, and exposure conditions. A framework for Bayesian model updating with structural reliability methods of concrete structures is presented and validated based on the results of an experimental campaign on prestressed concrete (PC) bridge deck beams extracted from a dismantled 50-year-old viaduct. Subset simulation techniques are applied for structural reliability analysis. The prior knowledge of concrete mechanical properties and residual prestressing levels is updated based on the results of laboratory tests. The statistical residual performance of the PC deck beams is compared with the experimental results of a full-scale load test up to collapse. The structural modelling is developed with bi-dimensional finite elements for plane-stress analysis formulated in accordance with the Modified Compression Field Theory (MCFT). The comparison between experimental and numerical results allows to validate the modelling strategies, the outcomes of the experimental tests, and the Bayesian updating framework.

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Bayesian updating; subset simulation; concrete structures; nonlinear finite element analysis; full-scale load tests; experimental validation

Introduction

Civil engineering problems are generally associated with conservative assumptions and their solution must rely on predictions and evaluations based on imperfect knowledge of physical phenomena and models (Ang and Tang 2007). Uncertainties may arise from several sources, including the inherent random nature of physical quantities, imperfections of engineering models, finite samples of available data to inform statistical estimators, and predictions to make decisions. Therefore, engineers should deal with the intrinsic natural variability in observable physical phenomena (aleatory uncertainties) and inaccurate predictive models and/or lack of empirical data (epistemic uncertainties). However, in engineering problems, it might be difficult to determine the nature of uncertainties (Der Kiureghian and Ditlevsen 2009). Classification of uncertainties into aleatory or epistemic can be useful, since the effect of aleatory randomness leads to a probability, while epistemic randomness expresses uncertainty in the estimated probability

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(Ang and Tang 2007). Moreover, much of the aleatory uncertainties in civil engineering owing to randomness cannot be reduced. On the other hand, epistemic uncertainties, associated with imperfect knowledge of the real world, may be reduced by refining the prediction models and/or enhancing the quality and quantity of information and experimental data by means of expert judgments, inspection activities, laboratory tests, and monitoring (Frangopol, Strauss, and Kim 2008a).

In the assessment of the structural performance of existing systems, discrepancies between the specified design properties (e.g. material mechanical characteristics) and the actual in-situ properties can be significant due to variants and human errors in the construction phase. Material and structural properties (e.g. residual prestressing stress) may also significantly change during the system lifetime due to instantaneous and/or long-term variations associated with sudden damaging events, continuous ageing and deterioration processes, changes in loading scenarios and exposure conditions, amplifying in this way the effects of the involved uncertainties (Biondini and Frangopol 2016). In this context, life-cycle reliability analysis of existing structures requires the use of models involving a wide spectrum of uncertainties associated with modelling strategies, simplifying assumptions, and numerical computations (Beck and Katafygiotis 1998; Katafygiotis and Beck 1998; Zhang, Feissel, and Antoni 2011). To this purpose, numerical analysis, in-situ activities, and experimental tests can be adopted to assess the actual structural characteristics and behaviour accounting for uncertainties. In bridge engineering, visual inspection-based protocols of existing structures are essential to detect damage scenarios and represent the basis activity for condition and safety assessment. Diagnostic and in-situ experimental tests can be adopted for material quality control measures and to verify design assumptions. Moreover, the extensive use of sensor technology in bridges and infrastructural systems can increase the knowledge regarding their actual performance and support a risk-based maintenance and management decision-making process under uncertainty (Corotis, Hugh Ellis, and Jiang 2005; Frangopol, Strauss, and Kim 2008b).

The various sources of information and sets of data gathered during the system's lifetime may be combined to better predict future structural conditions. Moreover, available information may require to be updated as additional data are gained. A proper tool to combine uncertain available information is represented by the Bayesian approach which can be used to update uncertain prior probabilistic models with new data and increase the accuracy of predictions by combining different sources of uncertainties for the purpose of decision-making (Ang and Tang 2007; Benjamin and Cornell 1970). The basic idea of Bayesian updating is to model the unknown parameters of a statistical distribution as random variables. Therefore, the different sources of information associated with the estimation of these parameters can be properly combined.

Several applications of Bayesian updating of civil engineering models are reported in the literature, and several notable examples can be provided. Tang (1973) adopted the Bayesian approach where the distribution of structural flaws and imperfections in metal structures has been updated based on the results of inspection activities. The effects of inspection data, diagnostic tests, and repair activities on the life-cycle safety and reliability assessment of in-service concrete structures through the use of Bayesian analysis have been investigated in Thoft-Christensen and Sørensen (1987), Mori and

Ellingwood (1994), Enright and Frangopol (1999), Val, Stewart, and Melchers (2000), Ma et al. (2013), Faroz, Pujari, and Ghosh (2016), Schneider, Thöns, and Straub (2017), and Gu and Li (2022), among others. The role and value of data gathered with monitoring systems on the structural reliability are presented in Kamariotis, Chatzi, and Straub (2022). Bayesian updating for the time-dependent reliability prediction of concrete structures in marine environments based on visual inspections of crack width and chloride concentration is presented in Akiyama, Frangopol, and Yoshida (2010). A general approach for the development of prediction functions and a performance assessment procedure of structures based on monitored extreme data is proposed in Frangopol, Strauss, and Kim (2008b) and Strauss, Frangopol, and Kim (2008).

In this paper, a novel framework for Bayesian updating with structural reliability methods combining the results of nonlinear structural analysis and experimental tests is presented and applied to existing concrete structures through the use of subset simulation techniques. The proposed approach is validated by computing the posterior predictive distributions of concrete mechanical properties and residual prestressing level of prestressed concrete (PC) bridge deck beams based on laboratory tests. Prior distributions are based on data reported in the original design documentation. The residual structural capacity of the PC beams is investigated with nonlinear analysis based on bi-dimensional finite elements for a plane-stress model formulated in accordance with the Modified Compression Field Theory (MCFT). The outcomes of a full-scale load test up to collapse are compared with the statistical numerical results associated with prior and posterior predictive distributions to validate the modelling strategies, the outcomes of the experimental tests, and the Bayesian updating procedure. Based on the above, the main contributions of this paper include a probabilistic approach to incorporate Bayesian updating in the residual performance assessment of existing structures combining different sources of data and information from design documentation and experimental tests, and the validation of the proposed framework based on laboratory tests and full-scale load testing on 50-year-old PC bridge deck beams.

Bayesian statistical inference

Bayesian statistical inference is a powerful and robust tool for the combination of probabilistic models with new information and data. Moreover, when data are available from a structural response, the Bayesian framework can also be applied as an inverse problem to derive information about the input random variables (Straub, Papaioannou, and Betz 2016).

In Bayesian inference, the model parameters of a statistical distribution are assumed as random variables associated with probability density functions. Therefore, their prior uncertainty is represented with a probability distribution that can be updated based on new information from observed data and/or expert judgement through the use of Bayes' theorem to achieve a posterior distribution (Ang and Tang 2007). A set of random variables $\mathbf{X}$ is defined by parameters which are also modelled as random variables $\boldsymbol{\Theta}$ through the prior probability density function (PDF) $f'_{\Theta}(\boldsymbol{\theta})$ which represents the initial belief of the parameters. When new information, judgements, and data $\boldsymbol{d}$ become available they can be used to update the prior distribution by means of Bayesian inference and achieve posterior distribution $f''_{\Theta}(\boldsymbol{\theta})$ of the model parameters introducing the likelihood function $L(\boldsymbol{\theta}|\boldsymbol{d})$, as follows:

$$f''_{\Theta}(\boldsymbol{\theta}) = \frac{L(\boldsymbol{\theta}|\boldsymbol{d})\, f'_{\Theta}(\boldsymbol{\theta})}{\int_{\boldsymbol{\theta}} L(\boldsymbol{\theta}|\boldsymbol{d})\, f'_{\Theta}(\boldsymbol{\theta})\, d\boldsymbol{\theta}}. \tag{1}$$

The prior probability distribution of the model parameters expresses the uncertainties before new observations are gathered. This entity can be based on expert judgements, physical requirements, and prior information. Prior distributions are commonly categorised into different types, including informative and uninformative priors which express strong or weak beliefs about the parameters, respectively (Gelman et al. 2013). On the other hand, the likelihood function $L(\boldsymbol{\theta}|\boldsymbol{d})$ can be seen as the knowledge gained with data $\boldsymbol{d}$ as follows:

$$L(\boldsymbol{\theta}|\boldsymbol{d}) \propto P(\boldsymbol{d}|\Theta = \boldsymbol{\theta}). \tag{2}$$

Based on a set of n observations, under the assumption of statistical independence, the overall likelihood function can be evaluated based on a chain product of the likelihood functions associated with individual observations (Straub and Papaioannou 2015):

$$L(\boldsymbol{\theta}|\boldsymbol{d}) = \prod_{i=1}^{n} L_i(\boldsymbol{\theta}|d_i). \tag{3}$$

In deriving the posterior distribution, Equation (1) can be solved analytically only if the distributions of the underlying random variables and the corresponding parameters are associated with statistical conjugate distributions (Ang and Tang 2007). To overcome this limitation, the Bayesian updating procedure can be converted into an equivalent structural reliability problem (Straub and Papaioannou 2015) and posterior distribution is generally evaluated numerically.

Bayesian updating with structural reliability methods

Probability of failure

Structural reliability problems are generally defined with respect to demand from external loadings associated with natural and/or human-made hazards compared to resistance related to the ability of a system to withstand those actions (Der Kiureghian and Liu 1986; Ditlevsen and Madsen 1996; Freudenthal 1956; Melchers and Beck 2018; Rackwitz 2001; Thoft-Christensen and Baker 1982). A structure can be considered safe if demand is no larger than resistance, where these two entities can be defined at different levels (e.g. materials, cross-section, structural component, and system). More generally, the failure of a structural system can be defined in terms of a limit state function which represents design requirements associated with serviceability or ultimate limit states. A limit state function $g(\mathbf{x}) = 0$ represents the analytical boundaries of a serviceability or ultimate limit state condition of the structural system associated with the random variables vector $\mathbf{X}$ and vector of model parameters $\boldsymbol{\theta}$. This limit state function divides the sample space of the random variables into two subdomains: safe domain (i.e. $\Omega_S = \{g(\mathbf{x}) > 0\}$) and failure domain (i.e. $\Omega_F = \{g(\mathbf{x}) \leq 0\}$). Resistance and demand are affected by uncertainties. Therefore, structural reliability has to be evaluated in probabilistic terms and associated with a probability of failure P_F. The system probability of failure P_F with respect to a specific limit state condition $g(\mathbf{x}) = 0$ can be defined as the expected value of the conditional probability estimate with respect to parameters $\boldsymbol{\theta}$, as follows:

$$P_F = \int_{\boldsymbol{\theta}} \int_{\Omega_F} f_{\mathbf{X}|\Theta}(\mathbf{x}|\boldsymbol{\theta})\, f_{\Theta}(\boldsymbol{\theta})\, d\mathbf{x}\, d\boldsymbol{\theta} = \int_{\Omega_F} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}, \tag{4}$$

where $f_{\mathbf{X}|\Theta}(\mathbf{x}|\boldsymbol{\theta})$ corresponds to the conditional probability density function of $\mathbf{X}$ for given $\boldsymbol{\theta}$, $f_{\Theta}(\boldsymbol{\theta})$ is the probability density function of parameters $\boldsymbol{\theta}$ (i.e. prior $f'_{\Theta}(\boldsymbol{\theta})$ or posterior $f''_{\Theta}(\boldsymbol{\theta})$ PDF), and $f_{\mathbf{X}}(\mathbf{x})$ is the unconditional distribution of $\mathbf{X}$, generally denoted as predictive since it takes into account uncertainties in the parameters $\boldsymbol{\theta}$ (Melchers and Beck 2018). It is worth noting that the probability of failure computed in Equation (4) includes both the natural variability of the underlying random variables and the uncertainty in the distribution parameters through the computation of the unconditional distribution of $\mathbf{X}$. However, in some cases, it might be useful to maintain the distinction between the two types of uncertainty. Separating the individual parts of inherent variability (aleatory uncertainty) and lack of knowledge (epistemic uncertainty) can display their contribution to the model output and help in resource allocation for data collection, since only epistemic uncertainty can be reduced (Faber 2005; Nannapaneni and Mahadevan 2016; Sankararaman and Mahadevan 2013). Methods to quantify the individual contributions of natural variability and parameter uncertainty in the random variables $\mathbf{X}$, in the output of a computational model, and in the structural reliability assessment generally refer to sensitivity analysis (Guo and Du 2007). In this paper, the computation of the probability of failure is performed in the Bayesian updating framework through the analogy with a structural reliability problem.

The evaluation of the probability of failure based on Equation (4) can be critical since the involved PDFs might be difficult to be modelled in a close form and/or the limit state function may be difficult to be formulated, especially in problems involving a large set of uncertainties. Analytical methods for solving the integral of Equation (4) include the first- and second-order reliability methods which are based on the approximation of the limit-state function by means of a first- or second-order Taylor expansion, respectively. These methods provide an approximate solution to the reliability problem (Ditlevsen and Madsen 1996; Fiessler, Neumann, and Rackwitz 1979; Hohenbichler and Rackwitz 1982). Additionally, the probability of failure can be evaluated numerically through simulation techniques. A recent historical review of structural reliability methods and numerical simulation techniques can be found in Ellingwood et al. (2024). Surrogate and machine learning methods have been also adopted in recent years for modelling the system limit state function with its surrogate and trying to reduce the complexity and computational cost of the associated reliability analysis problem (Echard, Gayton, and Lemaire 2011; Teixeira, Nogal, and O'Connor 2021).

Simulation methods

The main challenges of simulation methods in reliability assessment problems are associated with (a) evaluation of small probabilities of failure, (b) high dimensionality of the problems (i.e. large number of input random variables), and (c) complexity in the input-output relationship, i.e. complexity in the limit state function evaluation (Au and Patelli

2016). Several numerical simulation methods have been proposed in the literature. The traditional Monte Carlo simulation (MCS), originally presented in Metropolis and Ulam (1949), represents a simple and robust approach for the estimate of the probability of failure regardless of the complexity of the problem but is not computationally efficient in estimating small probabilities, which typically characterize ultimate state conditions of civil engineering problems. The Latin Hypercube Sampling (LHS) is an alternative numerical method proposed by McKay, Beckman, and Conover (1979) that introduces a stratification of the basic random variables space into intervals to spread the samples over the entire sampling space and therefore tend to reduce the size of the required sample data. In order to overcome the inefficiency of MCS in estimating small probabilities of failure, Importance Sampling (IS) techniques (Melchers 1989) have been developed to shift the limit state function, through an appropriate choice of an importance sampling distribution, towards the failure domain to generate samples from rare events more efficiently (Au and Beck 2003a). A major drawback of this method is associated with the difficulty of the selection of an appropriate IS distribution when a large number of basic random variables is involved (Au and Beck 2003a). Additionally, the subset simulation (SuS) method has been proposed in Au and Beck (2001) particularly for estimating small probabilities of failure through the generation of samples which explore efficiently the failure region (Au and Wang 2014). In this paper, due to its computational efficiency, SuS is briefly presented and adopted in the Bayesian updating based on structural reliability methods.

Subset simulation

The basic idea of SuS is to express the probability of failure as a product of larger conditional failure probabilities by introducing intermediate failure events (Au and Beck 2001). The failure domain $\Omega_\mathrm{F} = \{g(\mathbf{x}) \leq 0\}$ is the intersection of m intermediate nested domains Z_i, where $Z_0 = \mathbb{R}^M \supset Z_1 \supset \ldots \supset Z_\mathrm{m} = \Omega_\mathrm{F}$. The probability of failure is formulated through subset domains, $Z_i = \{g(\mathbf{x}) \leq b_i\}$ with $b_0 = \infty > b_1 > \ldots > b_m = 0$, as the product of a sequence of conditional probabilities:

$$P_F = P(\Omega_\mathrm{F}) = \prod_{i=1}^{m} P(Z_i|Z_{i-1}). \tag{5}$$

The samples are therefore adaptively simulated to gradually populate the rare-event region (Au and Beck 2003b). It is worth noting that, even if the probability of failure P_F is small, the appropriate selection of intermediate failure events Z_i can make the conditional probabilities $P(Z_i|Z_{i-1})$ sufficiently large to be easily estimated. Therefore, the problem of evaluating rare events in the original probability space is replaced by a sequence of more frequent events in the conditional probability space (Au and Beck 2001).

The first conditional probability $P(Z_1|Z_0) = P(Z_1)$, where Z_0 is the certain event, can be addressed by means of MCS. The other conditional probabilities $P(Z_i|Z_{i-1})$ require the generation of conditional samples that can be achieved by a Markov Chain Monte Carlo (MCMC) algorithm, which represents a powerful method for generating samples conditional on the failure domain Z_i ($i = 1, \ldots, m$-1). The choice of the intermediate failure events $Z_i = \{g(\mathbf{x}) \leq b_i\}$ is crucial in SuS since affects its efficiency. This is done by a proper selection of the limit state values b_i which are computed during the simulation process in order to obtain a predefined probability p_0 associated with a not-rare event and selected in order to avoid generating a large number of intermediate subsets m (Au and Beck 2001; Zuev et al. 2012).

It has been verified with numerical examples that SuS provides higher computational efficiency in estimating small probabilities of failure compared with crude MCS and other simulation methods (Au, Ching, and Beck 2007; Schuëller and Pradlwarter 2007). Moreover, the performance of SuS is not directly dependent on the number of input random variables (Schuëller, Pradlwarter, and Koutsourelakis 2004). The SuS method starts with the application of MCS to generate independent and identically distributed samples from the distribution of the input random variables. Then, an MCMC algorithm is adopted to generate samples conditional on the intermediate failure event Z_i ($i = 1, \ldots, m$-1) using as seeds the samples conditional on $Z_{i\text{-}1}$. Therefore, the efficiency of SuS is also associated with the ability of the adopted MCMC method to estimate the conditional probabilities through a proper number of samples (Papaioannou et al. 2015). The final conditional probability $P(Z_m|Z_{m-1})$ is given by the ratio of the number of samples for which $g(\mathbf{x}) \leq 0$ over the total number of simulated samples conditional on $Z_{m\text{-}1}$. In this final step, a larger number of generated samples, with respect to the intermediate events, can be used to properly populate the failure domain Ω_F (Straub and Papaioannou 2015).

Markov chain Monte Carlo

MCMC generates samples of a target distribution through a Markov chain whose stationary distribution is the target distribution (Gilks, Richardson, and Spiegelhalter 1995). In SuS, MCMC is adopted to generate samples in the subset Z_i based on the seeds that fail in the previous subset $Z_{i\text{-}1}$. When the simulated Markov chain reaches the stationary state, the generated samples are identically distributed, according to the conditional PDFs of the intermediate failure events, but not independent, due to the correlation of the Markov process. In fact, the coefficient of variation (i.e. CoV) of the estimates of the conditional probabilities is generally larger than the one of the MCS estimate with independent and identically distributed samples (Papaioannou et al. 2015). The Metropolis–Hastings (M-H) algorithm is the most popular class of MCMC algorithm (Hastings 1970; Metropolis et al. 1953), which is based on two steps. In the first step, starting from the samples that fell in subset Z_i (seeds), it generates a candidate sample from a proposal distribution with an acceptable probability for the next state of the Markov chain. In the second step, the sample is accepted if it lies in the domain Z_i, rejected otherwise (Beck and Au 2002). Additional methods have been also proposed starting from the M-H approach. These methods mainly differ in the choice of the type and spread of the proposal distribution. As an example, the conditional sampling in U-space (i.e. standard normal space), proposed in Papaioannou et al. (2015) and adopted in this paper within the SuS, is an MCMC algorithm in which the candidate sample always differs from the current state sample and the spread of the proposal distribution is modified adaptively during the simulation process to improve the efficiency of the Markov chain.

Bayesian updating

The approach proposed by Straub and Papaioannou (2015) allows for the interpretation of the Bayesian updating as a structural reliability problem introducing a uniform random variable U valid in the range [0; 1] to the space of the model parameters $\mathbf{\Theta}$. Defining the domain of the corresponding reliability problem $\Omega = [u \leq c \cdot L(\boldsymbol{\theta}|\boldsymbol{d})]$, where c is a constant term such that $c \cdot L(\boldsymbol{\theta}|\boldsymbol{d}) \leq 1$, Equation (1) can be rewritten as follows:

$$f''_{\mathbf{\Theta}}(\boldsymbol{\theta}) = \frac{\int_{u\in\Omega} f'_{\mathbf{\Theta}}(\boldsymbol{\theta})\, du}{\int_{[\boldsymbol{\theta},u]\in\Omega} f'_{\mathbf{\Theta}}(\boldsymbol{\theta})\, du\, d\boldsymbol{\theta}}, \tag{6}$$

where the denominator of Equation (6) represents a structural reliability problem associated with the limit state function $g(\boldsymbol{\theta}, u) = u - c \cdot L(\boldsymbol{\theta}|\boldsymbol{d}) = 0$ and it follows that the samples generated from the prior distribution $f'_{\mathbf{\Theta}}(\boldsymbol{\theta})$ that fall into the domain Ω are distributed according to the posterior distribution $f''_{\mathbf{\Theta}}(\boldsymbol{\theta})$ (Papaioannou et al. 2015). In the context of Bayesian updating based on structural reliability methods, a failure event (with respect to a structural reliability problem) can be associated with an observation event (with respect to Bayesian updating) to generate samples from the posterior distribution (Straub, Papaioannou, and Betz 2016). Therefore, a key advantage of this analogy is that Bayesian updating can be combined with any structural reliability method (Betz et al. 2018).

Once the probability density function of the model parameters has been updated to the posterior distribution $f''_{\mathbf{\Theta}}(\boldsymbol{\theta})$, it is possible to obtain the predictive (or expected) distribution of the underlying random variables $\mathbf{X}$ by means of the total probability theorem, as follows:

$$f_{\mathbf{X}}(\mathbf{x}) = \int_{\boldsymbol{\theta}} f_{\mathbf{X}|\mathbf{\Theta}}(\mathbf{x}|\boldsymbol{\theta})\, f''_{\mathbf{\Theta}}(\boldsymbol{\theta})\, d\boldsymbol{\theta}. \tag{7}$$

This represents the probability distribution for unobserved or future data and includes two types of uncertainties: (a) the remaining uncertainty about the parameters $\boldsymbol{\theta}$ after the observations have been made; (b) the random variation of the variables $\mathbf{X}$. The updated models are expected to provide more accurate response predictions to future analysis. Therefore, the Bayesian framework allows to explicitly consider measurement errors and can provide information on the accuracy of the updated model which remains uncertain (Straub and Papaioannou 2015).

Residual structural capacity assessment based on experimental tests

The effectiveness of the Bayesian updating in the residual performance assessment of existing concrete structures is investigated based on the results of an ongoing experimental campaign conducted on 50-year-old PC bridge deck beams. The results of laboratory tests on concrete mechanical properties and residual prestressing level are adopted to update prior information based on the available design documentation. The outcomes of a full-scale load test up to collapse are compared with the statistical numerical results associated with prior and posterior predictive distributions.

BRIDGE|50 Research Project

During the last decades, life-cycle-oriented criteria and methods have been developed for the safety and reliability assessment of structures under time-variant structural performance and uncertainties. Despite the relevant advances and accomplishments achieved in life-cycle structural engineering, incorporation and implementation in professional practice of life-cycle methods still need robust validation and accurate calibration based on experimental evidence and information gathered from existing structures and infrastructure facilities. In fact, deterioration and vulnerability models are very sensitive to changes in the key damage parameters and robust validation and accurate calibration are difficult tasks because of the limited availability of data (Biondini and Frangopol 2018). Based on this need, the BRIDGE|50 research project has been established jointly by Politecnico di Milano and Politecnico di Torino as part of an agreement with public authorities and private companies for a wide experimental campaign to investigate the residual structural performance of a 50-year-old 80-span double-deck road viaduct located in Turin, Italy (Biondini, Manto et al. 2021; Biondini, Tondolo et al. 2021). The multi-span simply supported grillage bridge deck was formed by precast PC beams, including ten inner I-beams and two lateral U-box beams, with a top cast-in-situ reinforced concrete (RC) slab. The demolition of the infrastructure after a lifetime of 50 years due to urban redevelopment and sub-services renewal allowed the investigation of typical bridge elements exposed to ageing and deterioration (Carsana et al. 2022; Carsana, Redaelli, and Biondini 2023) and was an opportunity to dismantle, preserve, and test several structural members, including 29 PC deck beams (25 I-beams and four U-box beams) and two PC pier caps.

PC bridge deck beams

The dismantled I-beams (Figure 1) are characterised by a total length of about 19.50 m and have a composite cross-section made of a precast PC I-beam with twenty 7-wire prestressing steel strands and a top cast-in-situ RC slab (Figure 2). The PC I-beam cross-section is characterised by main dimensions 58×90 cm (width and depth, respectively) and web thickness of 16 cm. The dimensions of the cross-section of the top RC slab are

(a) (b)

Figure 1. Views of PC bridge deck beams in the testing site: (a) Detail of a group of beams; (b) PC beam with (left) and without (right) top slab.

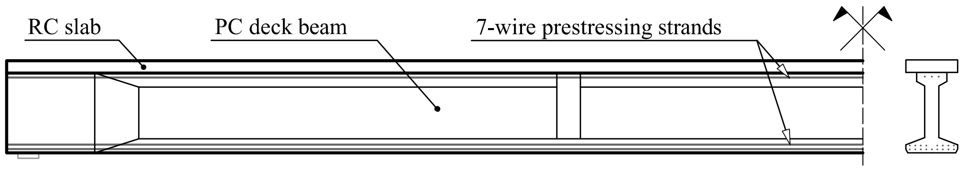

Figure 2. PC bridge deck beams: Longitudinal profile and midspan cross-section.

58 × 14 cm (width and depth, respectively). The prestressing strands are straight along the beam axis and characterised by a nominal diameter of 12.7 mm (effective area of 99 mm^2). The initial prestressing stress, net of instantaneous and estimated long-term losses reported in the technical design documentation is $\sigma_{pd} = 836$ MPa. Stirrups with a diameter of 8 mm and spacing of about 250 mm for the I-shaped cross-section have been evaluated based on pacometer diagnostic tests.

Laboratory tests for mechanical characterisation of materials

The actual concrete compressive strength and elastic modulus have been estimated by means of both non-destructive tests (i.e. rebound hammer tests and ultrasonic tests) and laboratory tests carried out on several cylindrical specimens extracted from the beams (Anghileri et al. 2023). The tensile concrete strength has been also evaluated through splitting tensile strength tests. In this paper, the results of laboratory tests only are considered since they are generally characterised by larger accuracy with respect to non-destructive techniques due to limited instrumentational and operational errors. Table 1 shows the overall sample mean and sample CoV evaluated on sets of *n* laboratory experimental tests carried out on samples extracted from a group of three PC beams. The actual concrete material properties resulted lower with respect to the design value reported in the original technical documentation (Savino et al. 2023). The material properties of prestressing steel have been also evaluated with laboratory tests. The results of tensile tests performed on eight strand samples extracted from the PC beams are in good agreement with the data reported in the design tests documentation (Savino et al. 2023).

Residual prestressing

The residual prestressing level in existing concrete systems represents key information for the performance assessment procedure due to its significant influence on early stage deflections and the development of concrete cracking. Several experimental techniques have been proposed in the literature for the evaluation of the effective prestressing level of in-service structures affected by instantaneous and/or long-term

Table 1. Sample mean $\bar{x}$ and coefficient of variation (CoV) of material properties estimated from *n* laboratory tests.

Property	n	Sample mean $\bar{x}$	CoV
Compressive concrete strength, f_c	25	32.34 MPa	0.14
Tensile concrete strength, f_{ct}	9	3.36 MPa	0.14
Concrete elastic modulus, E_c	8	27.28 GPa	0.08
Residual prestressing stress, σ_p	15	582 MPa	0.17

prestressing losses and steel corrosion effects (Osborn et al. 2012). The saw-cut method and the strand-cutting approach, based on the measurement of the strain on an isolated small concrete block or a cut prestressing strand, respectively, have been applied to the investigated beams (Savino 2023). Table 1 shows the overall sample mean and sample CoV evaluated from n values obtained from the strand-cutting method applied to a group of three beams. It is worth noting that the actual prestress level which resulted lower of about 25–35% with respect to the design value (i.e. $\sigma_{pd} = 836$ MPa) has been indirectly validated by means of nonlinear finite element analysis based on a comparison between numerical and experimental results (Anghileri and Biondini 2022, 2023).

Bayesian updating of concrete mechanical properties and residual prestressing

The diagnostic outcomes obtained from the PC deck beams (Table 1) are used to perform a probabilistic analysis based on Bayesian inference to update prior information associated with the available design documentation. In particular, the results of laboratory tests on concrete samples are used to update the initial knowledge of concrete mechanical properties in terms of compressive strength f_c, tensile strength f_{ct}, and elastic modulus E_c. Moreover, the results of the residual prestressing tests are used to gain accuracy in the estimate of the actual prestress σ_p with respect to the prior knowledge associated with data reported in the design documentation.

The investigated random variables (i.e. $\mathbf{X} = [f_c\ f_{ct}\ E_c\ \sigma_p]^T$) are assumed as non-negative truncated normally distributed, with both mean μ_X and standard deviation σ_X unknown (i.e. $\mathbf{\Theta} = [\mu_X\ \sigma_X]^T$). Statistical independence between the two parameters (μ_X, σ_X) of each random variable is assumed. The prior distribution of the parameters $f'_{\Theta}(\boldsymbol{\theta})$ is calibrated based on the data reported in the original design documentation with assumed probabilistic distributions given in Table 2.

The results of the laboratory tests given in Table 1 are samples of the corresponding random variables and can be adopted to update the prior probability distributions. Therefore, considering a single experimental outcome d_i of the generic random variable X, the likelihood function L_i for learning μ_X and σ_X is the conditional probability density function $f_X(x \mid \mu_X, \sigma_X)$ evaluated at $x = d_i$, i.e. $L_i(\mu_X, \sigma_X \mid d_i) = f_X(d_i \mid \mu_X, \sigma_X)$. Moreover, assuming statistical independence of the n experimental outcomes collected in vector $\boldsymbol{d}$, the combined likelihood function is evaluated based on Equation (3), as follows:

$$L(\mu_X, \sigma_X|\boldsymbol{d}) = \prod_{i=1}^{n} L_i(\mu_X, \sigma_X|d_i). \tag{8}$$

To visualise the effects of multiple observations on the likelihood function, the profile-likelihood functions are evaluated considering alternatively one of the two parameters (μ_X

Table 2. Prior distributions of mean μ_X and standard deviation σ_X of the underlying random variable X.

Random variable X ~ Truncated positive normal (μ_X, σ_X)	Prior distribution μ_X ~ Lognormal (μ, σ)	Prior distribution σ_X ~ Gamma (α, β)
Compressive concrete strength, f_c	$\mu = 40.24$ MPa; CoV = 0.15	$\alpha = 5$; $\beta = 1.0$
Tensile concrete strength, f_{ct}	$\mu = 3.04$ MPa; CoV = 0.10	$\alpha = 5$; $\beta = 10.0$
Concrete elastic modulus, E_c	$\mu = 33.41$ GPa; CoV = 0.15	$\alpha = 6$; $\beta = 1.0$
Residual prestressing stress, σ_p	$\mu = 836$ MPa; CoV = 0.20	$\alpha = 8$; $\beta = 0.1$

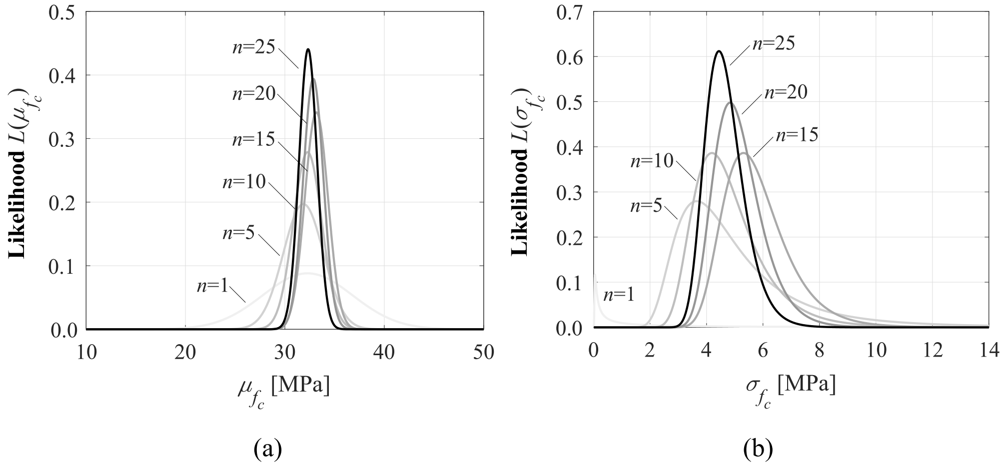

Figure 3. Profile-likelihood functions (scaled by $\left[\int L(\theta)d\theta\right]^{-1}$ with the integral defined over the entire real line) for different number n of samples of the parameter θ equals to (a) mean μ_X with fixed parameter $\sigma_X = \mathrm{s}$ and (b) standard deviation σ_X with fixed parameter $\mu_X = \bar{\mathrm{x}}$ for the random variable associated with concrete compressive strength ($X = f_c$).

or σ_X) fixed to a specific value, for example equal to the sample mean $\bar{\mathrm{x}}$ (i.e. $\mu_X = \bar{\mathrm{x}}$) or the sample standard deviation s (i.e. $\sigma_X = \mathrm{s}$). Figure 3 shows the evolution of the profile-likelihood functions associated with the first 1, 5, 10, 15, 20, and 25 samples for both mean μ_X with fixed parameter $\sigma_X = \mathrm{s}$ (Figure 3a) and standard deviation σ_X with fixed parameter $\mu_X = \bar{\mathrm{x}}$ (Figure 3b) of the concrete compressive strength (i.e. $X = f_c$). It can be observed how the increased number of experimental results tends to reduce the spread of the likelihood function.

The Bayesian updating approach in the context of the structural reliability method with the use of SuS is performed with a number of simulations $N = 10^4$ for each subset, to combine the prior distribution and the likelihood function according to Equation (6) and numerically evaluate the posterior distribution of the parameters. To visualize the generation of samples through the use of SuS, Figure 4 shows the joint samples of mean μ_X and uniform random variate U (Figure 4a) and the joint samples of mean μ_X and standard deviation σ_X (Figure 4b) of the residual prestressing stress (i.e. $X = \sigma_p$). The first subset, associated with independent and identically distributed samples, is generated by means of MCS. The other two subsets required the generation of conditional samples based on MCMC by means of a computation of the intermediate limit states b_i identified to obtain a target probability p_0 equals to 0.1 for each subset, which has been suggested for good efficiency (Au and Beck 2001).

Figures 5–8 show the prior distribution, profile-likelihood function, and posterior empirical frequency of both parameters (i.e. mean μ_X and standard deviation σ_X) of concrete compressive strength (Figure 5), concrete tensile strength (Figure 6), concrete elastic modulus (Figure 7), and residual prestressing stress (Figure 8). Once the Bayesian updating has been adopted to estimate the parameters of the probability distribution of a random variable X, the predictive posterior distribution can be evaluated by means of the total probability theorem according to Equation (7). To illustrate the effects of multiple experimental outcomes, Figure 9 shows the mean and standard deviation estimated

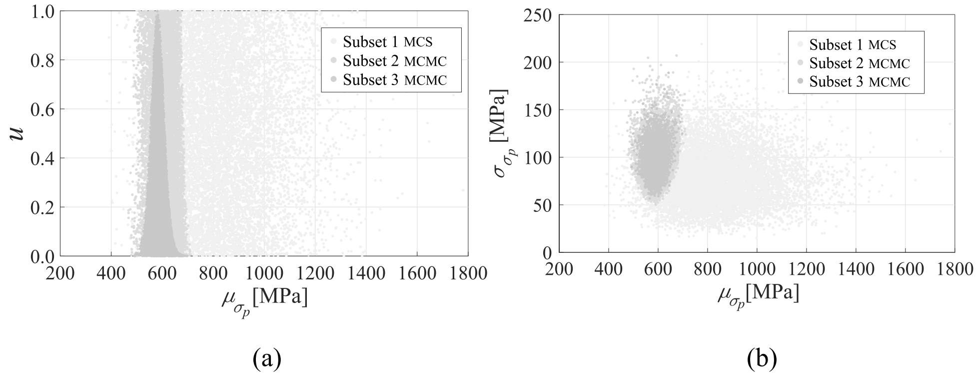

Figure 4. Bayesian updating based on subset simulation (10^4 samples for each subset) of residual prestressing stress ($X = \sigma_p$): (a) Joint samples mean μ_X and uniform random variate U; (b) Joint samples mean μ_X and standard deviation σ_X.

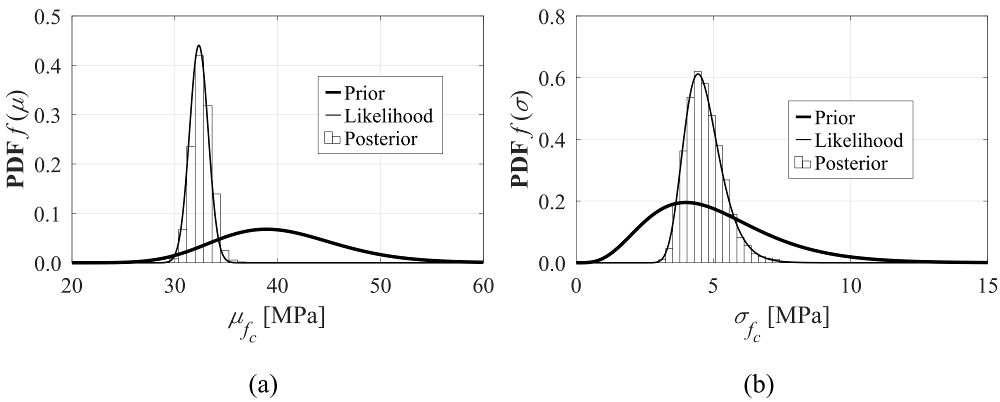

Figure 5. Prior distribution, profile-likelihood function, and posterior empirical frequency of (a) mean μ_X and (b) standard deviation σ_X of concrete compressive strength ($X = f_c$).

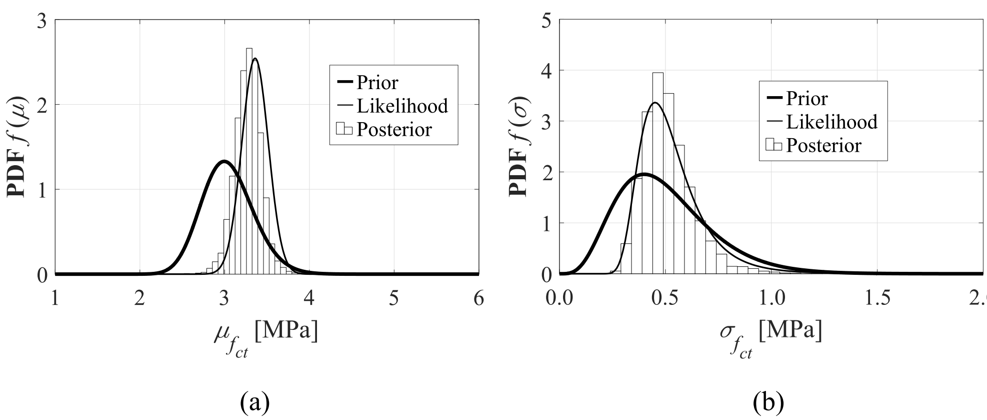

Figure 6. Prior distribution, profile-likelihood function, and posterior empirical frequency of (a) mean μ_X and (b) standard deviation σ_X of concrete tensile strength ($X = f_{ct}$).

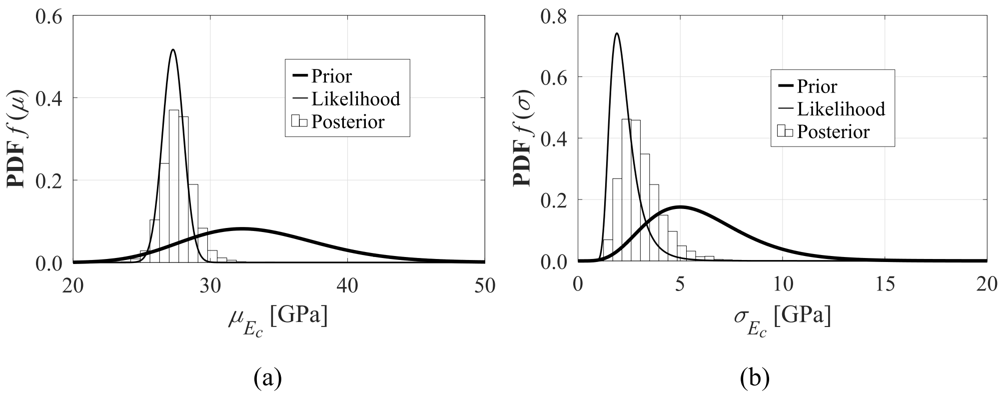

Figure 7. Prior distribution, profile-likelihood function, and posterior empirical frequency of (a) mean μ_X and (b) standard deviation σ_X of concrete elastic modulus ($X = E_c$).

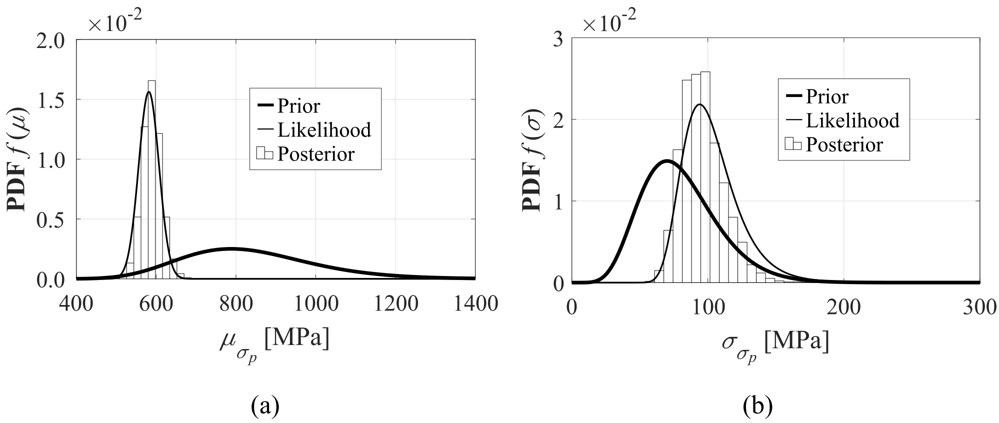

Figure 8. Prior distribution, profile-likelihood function, and posterior empirical frequency of (a) mean μ_X and (b) standard deviation σ_X of residual prestressing stress ($X = \sigma_p$).

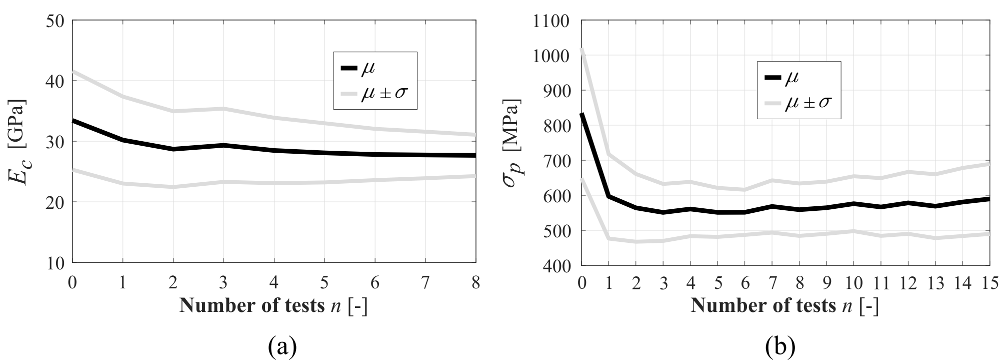

Figure 9. Mean μ and standard deviation σ estimated from posterior empirical frequency of (a) concrete elastic modulus E_c and (b) residual prestressing stress σ_p, versus the number n of experimental tests.

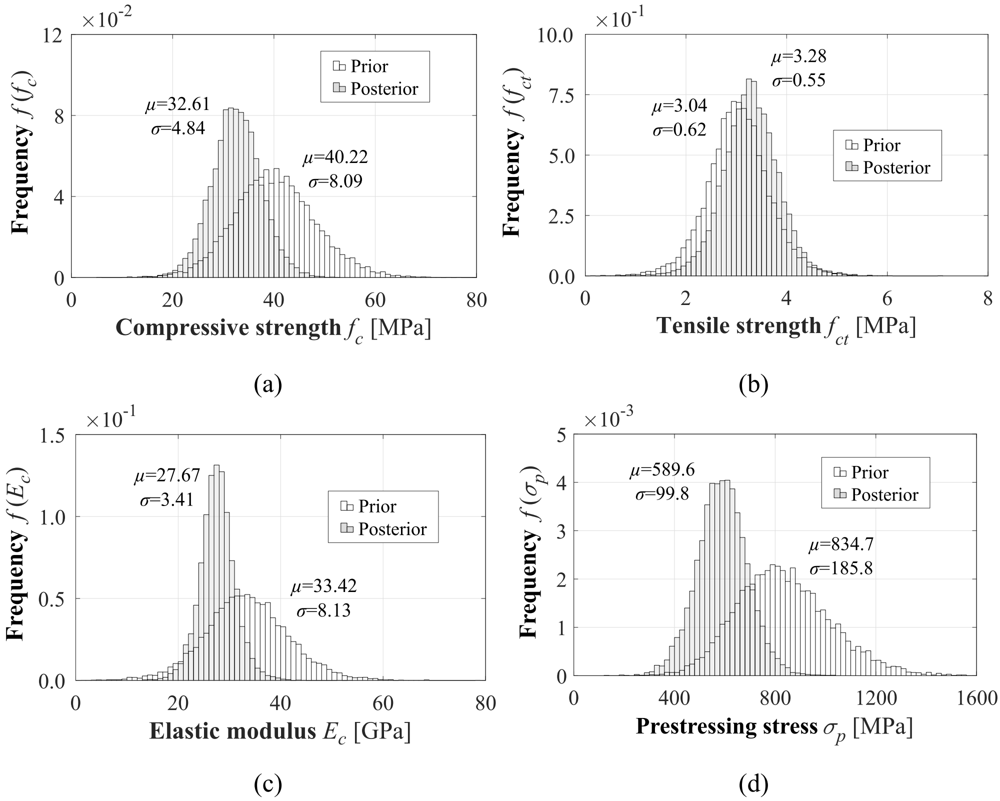

Figure 10. Prior and posterior predictive empirical frequencies: (a) Concrete compressive strength f_c; (b) Concrete tensile strength f_{ct}; (c) Concrete elastic modulus E_c; and (d) Residual prestressing stress σ_p.

from the posterior empirical frequency of concrete elastic modulus (Figure 9a) and residual prestress (Figure 9b) based on different numbers n of samples from the laboratory tests. Figure 10 shows both prior predictive and posterior predictive empirical

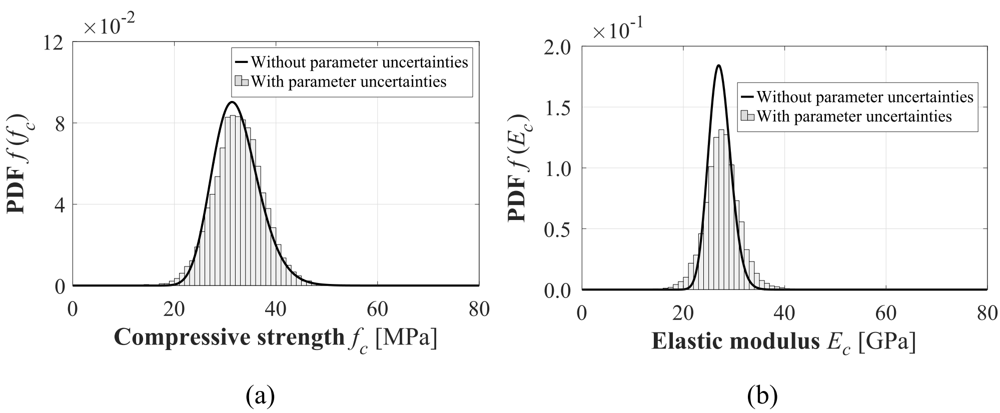

Figure 11. Posterior empirical frequency versus assumed lognormal PDF without parameter uncertainties ($\mu_X = \bar{x}$; $\sigma_X = s$) of (a) concrete compressive strength f_c and (b) concrete elastic modulus E_c.

frequencies of concrete compressive strength (Figure 10a), concrete tensile strength (Figure 10b), concrete elastic modulus (Figure 10c), and residual prestressing stress (Figure 10d). It can be noted that the mean value and the standard deviation of the posterior predictive distribution of the random variables tend to the sample mean and sample standard deviation of the laboratory outcomes (Table 1), particularly when a large number of experimental tests is used. In order to visualize the influence of the parameter uncertainties (i.e. statistical uncertainty), Figure 11 shows the comparison between the posterior empirical frequency diagram and the PDF of an assumed statistical model without parameter uncertainties. In particular, statistically independent lognormal distributions to model both the concrete compressive strength (i.e. $X = f_c$) and the concrete elastic modulus (i.e. $X = E_c$) have been assumed with mean $\mu_X = \bar{x}$ and standard deviation $\sigma_X = s$ based on the results of laboratory tests (Table 1). These models do not take into account the uncertainty associated with the model parameters μ_X and σ_X. Therefore, they do not consider the fact that sample mean $\bar{x}$ and sample standard deviation s have been estimated from a finite sample. It can be seen that for both concrete compressive strength (Figure 11a) and concrete elastic modulus (Figure 11b) the parameter uncertainties provide higher weight to the tails of the posterior empirical frequency diagram, as also indicated by similar comparisons (Jacinto, Neves, and Santos 2016), particularly for the case of the lower number of experimental tests. These results can also be used to plan additional experimental tests to reduce the parameter uncertainties.

Full-scale load tests

The residual structural capacity of the PC bridge deck beams is investigated by means of full-scale load tests up to failure using the testing framework shown in Figure 12 (Savino et al. 2023; Tondolo et al. 2021, 2022). The beams are tested under simple supports with span length of about 19.00 m and loaded by means of two transverse steel beams. Experimental sensors based on load cells, transducers, and displacement potentiometers allowed recording during the tests several quantities, including applied load, bending and shear strains, strand slips, support settlements, and vertical deflection (Tondolo et al. 2021, 2022).

(a) (b)

Figure 12. Full-scale load tests: (a) Lifting of a beam over the testing frame; (b) View of the testing framework with the PC deck beam without RC slab during the test.

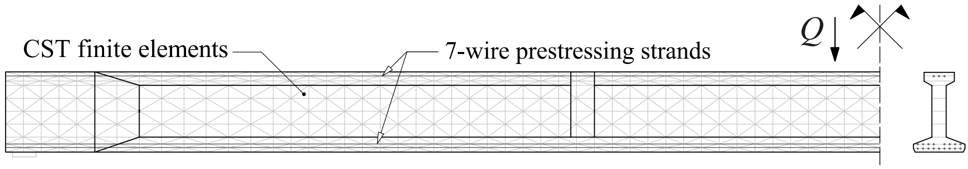

Figure 13. Structural modelling and finite element discretisation of PC bridge deck beam without top RC slab (MCFT).

Multiple load tests with different shear span ratios $\alpha = a/l$ (i.e. a = shear span; l = half beam span) have been conducted to explore both bending and shear failures. Moreover, in order to investigate the structural capacity of the PC beam only, one of the beams has been tested under a three-point loading (i.e. $\alpha \approx 1.00$) up to failure after the removal of the top RC slab. The results of the experimental test carried on this PC beam without the top RC slab are considered in this paper for nonlinear finite element analysis, comparison of numerical and experimental results, and probabilistic analysis.

Structural modelling and nonlinear finite element analysis

The residual structural performance of the PC bridge deck beam without the top RC slab is evaluated based on a plane-stress finite element model formulated in accordance with the MCFT (Vecchio and Collins 1986). This approach is based on a smeared rotating crack model where cracks change orientation according to the direction of principal strains which is considered coincident with the direction of principal stresses. The cracked RC medium is assumed as an orthotropic material with its own constitutive laws. Equilibrium, compatibility, and constitutive laws are formulated in terms of average stresses and average strains. The MCFT has been selected based on multiple factors, including the robustness of the formulation, capability to describe the real structural behaviour, accuracy of the solution process, and computational cost (Vecchio 2001).

The structural modelling exploits the symmetry and is based on a discretisation of half beam with 940 constant strain triangle (CST) finite elements (Figure 13). The formulation is based on a displacement-based approach (Bontempi, Malerba, and Romano 1995; Malerba 1998). Stirrups are modelled as smeared reinforcement over the beam concrete volume. Longitudinal reinforcing steel bars and prestressing strands are modelled as truss elements attached to the finite element mesh. The behaviour of concrete in compression is based on a uniaxial stress–strain relationship in the principal directions described by Hognestad parabola with compressive strength related to the transversal principal strain to account for cracking effects. The behaviour of concrete in tension is assumed linear up to cracking with a post-cracking softening branch accounting for the tension stiffening effect. The behaviour of both reinforcing and prestressing steel is based on a bilinear hardening constitutive law.

Bayesian updating based on experimental results

The results of the full-scale load test are considered to investigate the role and effects of data and information gathered with laboratory tests and combined with prior knowledge

Table 3. Mean μ_X and coefficient of variation (CoV) of the random variable X associated with mechanical properties of reinforcing and prestressing steel.

Random variable $X \sim$ *Lognormal*	Mean μ_X	CoV
Reinforcing steel yielding strength, f_{sy}	450 MPa	0.20
Reinforcing steel ultimate strength, f_{su}	540 MPa	0.20
Reinforcing steel elastic modulus, E_s	200 GPa	0.20
Prestressing steel yielding strength, f_{py}	1522 MPa	0.05
Prestressing steel ultimate strength, f_{pu}	1763 MPa	0.03
Prestressing steel elastic modulus, E_p	195 GPa	0.10

by means of Bayesian inference. The prior and posterior predictive PDFs are adopted in the probabilistic nonlinear finite element analysis to evaluate the effects of Bayesian updating on the residual structural performance of the PC beam.

The mechanical properties of concrete (i.e. compressive strength, tensile strength, and elastic modulus) and the residual prestressing level are assumed based on the prior and posterior distributions of the Bayesian updating results (Figure 10). Moreover, the mechanical properties of both reinforcing and prestressing steel are assumed as statistically independent lognormally distributed random variables with parameters listed in Table 3. The mean value and CoV of prestressing steel mechanical properties have been calibrated based on experimental tests. The probabilistic analysis is carried out with MCS with 10^4 samples. To investigate the effects of data gathered during the laboratory tests, the outcomes of the full-scale load test are compared with the statistical

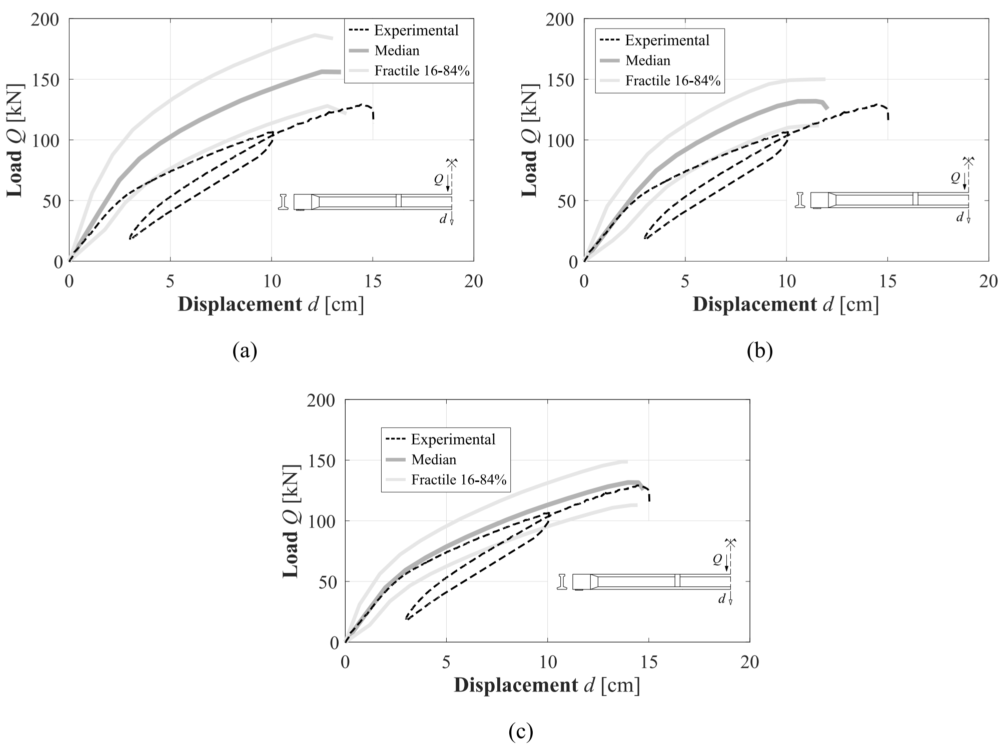

Figure 14. Load Q versus midspan displacement d of PC deck beam: Experimental results vs median, 16% and 84% fractiles of numerical results (MCFT): (a) Case (I); (b) Case (II); and (c) Case (III).

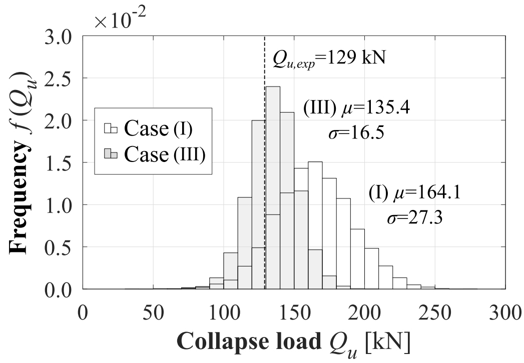

Figure 15. Empirical frequency of collapse load Q_u computed from nonlinear finite element analysis (MCFT) for Case (I) and Case (III).

numerical results associated with three different case studies. Case (I) assumes the concrete mechanical properties and the prestressing level associated with the prior predictive distributions of the Bayesian framework; Case (II) introduces the posterior predictive distribution of the concrete mechanical properties (Figures 10a, 10b, and 10c); Case (III) additionally accounts for the posterior predictive distribution of the estimated residual prestressing level (Figure 10d). Figure 14 shows the full-scale load test results, in terms of load Q versus midspan displacement d, of the PC deck beam comparing the experimental results and numerical (MCFT) probabilistic outcomes (i.e. median, 16% and 84% fractiles) for Case (I) (Figure 14a), Case (II) (Figure 14b), and Case (III) (Figure 14c). The experimental protocol of the full-scale load test is based on a preliminary loading phase up to $Q = 107$ kN reaching concrete cracking, a stop under loading for the assessment of the cracking pattern, subsequent unloading, and final reloading up to $Q = 129$ kN. The nonlinear finite element analysis is carried out under monotonic loading. The comparison of the results of the three case studies shows a significant reduction of the involved uncertainties. Additionally, Figure 15 shows the empirical frequency of the collapse load computed from nonlinear finite element analysis (MCFT) for Case (I) and Case (III). The close correspondence for Case (III) of statistical predictive outcomes with the experimental result allows to validate the finite element formulation, the results of the diagnostic activities, and the Bayesian updating procedure.

Conclusions

The role and effects of Bayesian updating with structural reliability methods within the residual performance assessment of concrete structures have been evaluated and discussed based on the comparison between statistical numerical results and the experimental outcomes of a wide experimental campaign on 50-year-old PC bridge deck beams. The Bayesian framework has been investigated by means of the subset simulation technique. The outcomes of laboratory tests on concrete material properties and residual prestressing level on the PC beams have been used to update the prior knowledge based on data reported in the original design documentation. Structural modelling has been developed by means of a bi-dimensional finite element for plane stress analysis formulated in accordance with the MCFT accounting for the material nonlinearities associated with constitutive laws of the materials, i.e. concrete, reinforcing steel, and prestressing steel. The effects of multiple experimental outcomes have been illustrated showing how the mean value and the standard deviation of the posterior predictive distribution of the updated random variables tend to the sample mean and sample standard deviation of the experimental results. Moreover, the outcomes of the performed analyses have shown the role and influence of parameter uncertainties. The good agreement between experimental and numerical results from the probabilistic analysis has allowed the validation of the modelling strategies, the outcomes of the laboratory tests, and the Bayesian updating framework. Overall, the results presented in this paper can contribute to updating the residual performance, safety, and reliability predictions of existing structures by means of experimental tests. In addition, the proposed approach can establish a solid ground to address additional experimental tests, further investigate the structural behaviour of the tested PC beams, and support the proper planning of the ongoing full-scale load tests. Future developments will be devoted to increasing the amount of experimental data and information to further investigate the probabilistic residual structural performance of the PC bridge deck beams.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

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