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# Lifetime reliability-based optimization of reinforced concrete cross-sections under corrosion

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ABSTRACT

This paper presents a lifetime reliability-based approach to the optimization of reinforced concrete (RC) cross-sections in an aggressive environment. The lifetime structural performance is evaluated by using a general methodology for time-variant analysis of RC structures subjected to diffusive attacks from aggressive agents with corrosion of the reinforcement. The lifetime probabilistic optimization is formulated at the cross-sectional level and is aimed to minimize the material cost under a time-dependent constraint on the structural reliability. The optimization problem is solved by combining a discrete gradient-based optimization method with a Monte Carlo simulation. The obtained results demonstrate that in a lifetime-oriented design the amount and location of the steel reinforcement and the value of the concrete cover play a crucial role for the optimal achievement of the desired lifetime structural performance.



## 1. Introduction

The design of concrete structures under chemical–physical damage phenomena is usually based on simple criteria associated with prescribed environmental conditions. Such criteria introduce threshold values for concrete cover, water-cement ratio, amount and type of cement, among others, to limit the effects of local damage due to carbonation of concrete and corrosion of reinforcement. However, the lifetime performance of concrete structures exposed to aggressive agents may significantly depend on the structural geometry and reinforcement layout. For this reason, a rational lifetime approach to optimum structural design of durable concrete structures should lead to find design solutions able to comply with the desired performance not only at the initial time of construction, but also during the expected lifetime by taking into account the effects induced by unavoidable sources of damage under uncertainty.

Based on these premises, life-cycle approaches to structural optimization of deteriorating systems have been proposed to highlight the fundamental role played by the time-variant performance in the optimal maintenance planning and selection of the optimum structural design [1–13]. Reliability-based life-cycle optimization approaches to deteriorating RC structures have been reported in [2,4,6,7,10]. A life-cycle approach to structural optimization has been recently developed in a deterministic context for truss and frame structures composed by homogeneous members [9,13], as well as for RC frames [11], by assuming material degradation laws associated with general deterioration processes. In this paper the deterministic formulation is extended to a probabilistic context to the minimum lifetime cost design of RC cross-sections subjected to diffusive attacks from environmental aggressive agents, like sulfate and chloride, which may cause a deterioration process with corrosion of the steel reinforcement.

The evolution of the lifetime structural performance is evaluated by using a general procedure proposed in previous works [14–16]. The lifetime probabilistic optimization is formulated at the cross-sectional level and is aimed to minimize the cost of the materials, concrete and steel, under a time-dependent constraint on the structural reliability. The optimization problem is solved by combining a discrete gradient-based optimization method with a Monte Carlo simulation. The role played by a lifetime approach to structural optimization is shown by comparing the optimal solutions obtained with a classical time-invariant formulation, which considers the initial undamaged state only, and the proposed lifetime formulation, where the time evolution of the structural performance is taken into account. The obtained results show that in a lifetime-oriented design the minimum feasible area of reinforcement is not associated with the maximum depth of the steel bars over the concrete cross-section, as expected in a classical time-invariant approach, but the amount and location of the reinforcement and the value of the concrete cover play a crucial role.

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## 2. Lifetime performance of RC structures

A lifetime performance analysis of RC structures in aggressive environments should be able to account for both the diffusion process of the aggressive agents, like sulfate and chloride, and the corresponding mechanical damage induced by diffusion, which usually involves corrosion of the reinforcement [17–19].

### 2.1. Cellular automata simulation of the diffusion process

In the proposed approach, the diffusion process of aggressive agents in concrete is described by the Fick's laws of diffusion [20] and reproduced by using a special class of evolutionary algorithms called cellular automata [21]. In its basic form, a cellular automaton consists of a regular uniform grid of cells with a discrete variable in each cell which can take on a finite number of states. During time, cellular automata evolve in discrete time steps according to a set of local evolutionary rules. For the diffusion problem in one-dimension, the discrete variable in the cell $i$ refers to the concentration $C_i^k = C(z_i, t_k)$ at point $z_i$ and time $t_k$, and the following evolutionary rule can be adopted [14]:

$$C_i^{k+1} = \phi_0 C_i^k + \frac{1-\phi_0}{2}\left(C_{i-1}^k + C_{i+1}^k\right) \tag{1}$$

where $\phi_0 \in [0;1]$ is a suitable evolutionary coefficient. To prove the equivalence between the evolutionary rule of the cellular automaton and the diffusion differential equation, the previous relationship can be rewritten in the following equivalent form [22]:

$$C(z, t+\Delta t) = \phi_0 C(z,t) + \frac{1-\phi_0}{2}[C(z-\Delta z, t) + C(z+\Delta z, t)] \tag{2}$$

which refers to a cellular automaton with grid dimension $\Delta z$ and time step $\Delta t$, and where $z_i = z$, $z_{i\pm1} = z \pm \Delta z$, $t_k = t$, and $t_{k+1} = t + \Delta t$. By subtracting $C(z, t)$ from both sides and dividing by $\Delta t$, we obtain:

$$\frac{C(z,t+\Delta t) - C(z,t)}{\Delta t} = \frac{1-\phi_0}{2}\frac{1}{\Delta t}[C(z-\Delta z,t) - 2C(z,t) + C(z+\Delta z,t)] \tag{3}$$

or:

$$\frac{C(z,t+\Delta t) - C(z,t)}{\Delta t} = \frac{1-\phi_0}{2}\frac{\Delta z^2}{\Delta t}\frac{[C(z+\Delta z,t) - C(z,t)] - [C(z,t) - C(z-\Delta z,t)]}{\Delta z^2} \tag{4}$$

where the right hand term has been multiplied and divided by $\Delta z^2$. By denoting:

$$D = \frac{1-\phi_0}{2}\frac{\Delta z^2}{\Delta t} \tag{5}$$

and taking the limit $\Delta z \to 0$ and $\Delta t \to 0$, the Fick's diffusion equation is finally obtained:

$$\frac{\partial C(z,t)}{\partial t} = D\frac{\partial^2 C(z,t)}{\partial z^2} \tag{6}$$

Therefore, the diffusion process can be simulated according to a prescribed value of the diffusivity $D$ by properly relating such parameter to the grid dimension $\Delta z$ and the time step $\Delta t$ of the automaton. This demonstration can be generalized to $d$-dimensions ($d$ = 1, 2, 3), where the diffusion equation:

$$D\nabla^2 C(\mathbf{z},t) = \frac{\partial C(\mathbf{z},t)}{\partial t}, \quad \nabla^2 = \sum_{j=1}^{d}\frac{\partial^2}{\partial z_j^2} \tag{7}$$

can be reproduced by using the following evolutionary rule:

$$C_i^{k+1} = \phi_0 C_i^k + \frac{1-\phi_0}{2d}\sum_{j=1}^{d}\left(C_{i-1,j}^k + C_{i+1,j}^k\right) \tag{8}$$

with:

$$D = \frac{1-\phi_0}{2d}\frac{\Delta z^2}{\Delta t} \tag{9}$$

Fig. 1 shows the pattern of cells involved in the evolutionary rule for a two-dimensional cellular automaton ($d$ = 2).

The value $\phi_0 = 1/2$ is generally a proper choice to obtain a good accuracy of the automaton. Morever, a deterministic description of the local diffusion mechanism usually allows to evaluate the global effects of the diffusion process with adequate accuracy for design purposes. However, it is worth noting that the stochastic effects in the mass transfer associated to the local random variability of material diffusivity $D$ can easily be taken into account by assuming $\phi_0$ as random variable. In this way, by adopting different probabilistic distributions for cracked and uncracked concrete, it is also possible to simulate local modifications in the rate of mass diffusion induced by cracking, which usually involve higher gradient of concentration and coupling effects between diffusion and damage [14].

### 2.2. Modeling of steel corrosion

Structural damage induced by diffusion may involve deterioration of concrete and corrosion of reinforcement [18]. This study will focus on the effects of corrosion on the structural performance. To this aim, a deterioration process with no damage of concrete and uniform corrosion is considered. As shown in [14], the percentage loss of steel resistant area for a corroded reinforcement bar can be effectively described by means of a dimensionless damage index $\delta_s$ which provide a direct measure of damage within the range [0;1]. The corrosion rate of steel depends on the concentration of the aggressive agent [19] and based on available data for sulfate and chloride attacks [23] a linear dependency can be approximately assumed. The damage index $\delta_{sm} = \delta_{sm}(\mathbf{z}_m, t)$ of a reinforcement bar $m$ located at point $\mathbf{z}_m = (z_{1m}, z_{2m})$ over the concrete cross-section is therefore correlated at each time instant $t$ to the diffusion process by assuming a linear relationship between the rate of damage and the mass concentration $C = C(\mathbf{z}_m, t)$ of the aggressive agent [14]:

$$\frac{\partial \delta_{sm}(\mathbf{z}_m, t)}{\partial t} = \frac{C(\mathbf{z}_m, t)}{C_s \Delta t_s} \tag{10}$$

where $C_s$ represents the value of constant concentration which lead to a complete damage ($\delta_s = 1$) after the time period $\Delta t_s$. Since corrosion does not occur until the accumulation of the aggressive agent

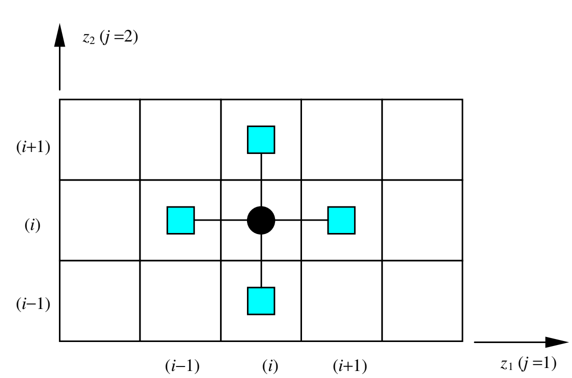


**Fig. 1.** Pattern of cells involved in the evolutionary rule of a two-dimensional cellular automaton.

at the bar surface exceeds a critical threshold of concentration $C_{cr}$ [24], the initial condition $\delta_{sm}(\mathbf{z}_m, t_{0m}) = 0$ with $t_{0m} = \max\{t \mid C(\mathbf{z}_m, t) \leqslant C_{cr}\}$ is assumed.

## 3. Lifetime approach to reliability-based structural optimization

### 3.1. Formulation of the optimization problem

The purpose of a lifetime design optimization process is to find a vector of design variables $\mathbf{x}$ which optimizes the value of an objective function $f(\mathbf{x})$, according to both side constraints with bounds $\mathbf{x}^-$ and $\mathbf{x}^+$, and inequality time-variant behavioral constraints $\mathbf{g}(\mathbf{x},t) \leqslant \mathbf{0}$ over a prescribed lifetime $T$. The deterministic lifetime design optimization problem can be formulated as follows:

$$\min_{\mathbf{x}\in D} f(\mathbf{x}) \quad D = \{\mathbf{x} | \mathbf{x}^- \leqslant \mathbf{x} \leqslant \mathbf{x}^+, \mathbf{g}(\mathbf{x},t) \leqslant \mathbf{0}, t \leqslant T\} \tag{11}$$

Reliability-based design is concerned with the evaluation of the probability of failure:

$$P_F(\mathbf{x},t) = P[\mathbf{g}(\mathbf{x},t) \leqslant \mathbf{0}], t \leqslant T \tag{12}$$

or the corresponding reliability index:

$$\beta(\mathbf{x},t) = -\Phi^{-1}[P_F(\mathbf{x},t)], t \leqslant T \tag{13}$$

where $\Phi = \Phi(\cdot)$ is the standard normal cumulative probability function. Therefore, the lifetime probabilistic design optimization problem can be formulated as follows:

$$\min_{\mathbf{x}\in D} f(\mathbf{x}) \quad D = \{\mathbf{x} | \mathbf{x}^- \leqslant \mathbf{x} \leqslant \mathbf{x}^+, \beta(\mathbf{x},t) \geqslant \bar{\beta}(t), t \leqslant T\} \tag{14}$$

where the lifetime target reliability $\bar{\beta} = \bar{\beta}(t)$ is in general time-variant since it reflects several factors which may change over time, including type and importance of the structure, possible failure consequences, warning of failure occurrence, and socio-economic criteria [25,26].

### 3.2. Lifetime optimization of RC cross-sections

Several quantities may be chosen as target requirements for the optimal design of RC cross-sections. Since no maintenance interventions are applied during the lifetime $T$, the objective function adopted in this study is related to the cost of the materials, concrete and steel:

$$f(\mathbf{x}) = A_c(\mathbf{x}) + \chi A_s(\mathbf{x}) \tag{15}$$

where $A_c(\mathbf{x})$ and $A_s(\mathbf{x})$ are the total area of concrete and steel, respectively, and $\chi = c_s/c_c$ is the ratio between the unit costs of steel, $c_s$, and concrete, $c_c$, respectively. Clearly, additional cost components, i.e. cost of formwork and so on, may be considered in the present formulation. However, it is worth noting that the selected objective function represents a consistent criterion to compare different design solutions rather than the actual structural cost in a strict sense.

By denoting $M_R = M_R(\mathbf{x},t)$ the resistant bending moment of the cross-section, and $M_A = M_A(\mathbf{x},t)$ the acting bending moment, a time-variant behavioral design constraint is set at the ultimate limit state in terms of safety factor $\Theta = \Theta(\mathbf{x},t)$ as follows:

$$\Theta(\mathbf{x},t) = \frac{M_R(\mathbf{x},t)}{M_A(\mathbf{x},t)} \geqslant 1, t \leqslant T \tag{16}$$

If a lognormal distribution can be selected as appropriate model for $M_R$ and $M_A$, with mean values $\mu_R$ and $\mu_A$ and coefficients of variation $\delta_R$ and $\delta_A$, respectively, the probabilistic distribution of the safety factor $\Theta$ is also lognormal with the following statistical parameters [27]:

$$\mu_\Theta = \frac{\mu_R}{\mu_A}(1+\delta_A^2) \quad \delta_\Theta = \sqrt{\delta_R^2 + \delta_A^2 + \delta_R^2\delta_A^2} \tag{17}$$

Based on this model, the time-variant reliability index can be computed as follows:

$$\beta(\mathbf{x},t) = \frac{\lambda_\Theta(\mathbf{x},t)}{\zeta_\Theta(\mathbf{x},t)} \tag{18}$$

where $\lambda_\Theta$ and $\zeta_\Theta$ are, respectively, the mean and standard deviation of the normal random variable $\ln \Theta$:

$$\lambda_\Theta = \ln \mu_\Theta - \frac{1}{2}\zeta_\Theta^2 \quad \zeta_\Theta^2 = \ln(1+\delta_\Theta^2) \tag{19}$$

For RC cross-sections the vector of design variables $\mathbf{x}$ usually includes continuous variables, related to the size and geometry of the cross-section, and integer variables, associated to the standardized steel bar diameters generally available for ordinary RC structures. In this study, the lifetime reliability-based optimization problem is solved by combining a generalized reduced gradient method and a branch-and-bound method [28] with a Monte Carlo simulation [29] applied at each step of the solution process to evaluate the time-variant statistical parameters $\mu_R$ and $\delta_R$ of the resistant bending moment $M_R$ over the lifetime $T$.

## 4. Application to a RC cross-section

The proposed formulation is applied in the following to the lifetime reliability-based optimum design of both geometrical dimensions and reinforcement layout of a rectangular cross-section under diffusive attack of aggressive agents.

### 4.1. Design model

The design model of the cross-section refers to the layout of the reinforcing steel bars shown in Fig. 2a. With reference to a width $b = 400$ mm and a bar spacing $s = 50$ mm, and denoting $h$ the heigth of the cross-section, $h^* = (h - c)$ the maximum bar depth, $c$ the concrete cover, and $\emptyset_{ij}$ the diameter of the reinforcing bar $(i,j)$, a vector $\mathbf{x} = [\mathbf{x}_1^T \quad \mathbf{x}_2^T]^T$ collecting a set of six design variables, including two non negative continuous variables $\mathbf{x}_1 = [h^* \quad c]^T$ and four non negative discrete variables $\mathbf{x}_2 = [\emptyset_1 \quad \emptyset_2 \quad \emptyset_3 \quad \emptyset_{34}]^T$, with $\emptyset_{ij} = \emptyset_i \; \forall j \leqslant 3$, is assumed. The optimal values of the design variables are searched considering the following side constraints:

$$h^*_{min} \leqslant h^* \leqslant h^*_{max} \tag{20}$$

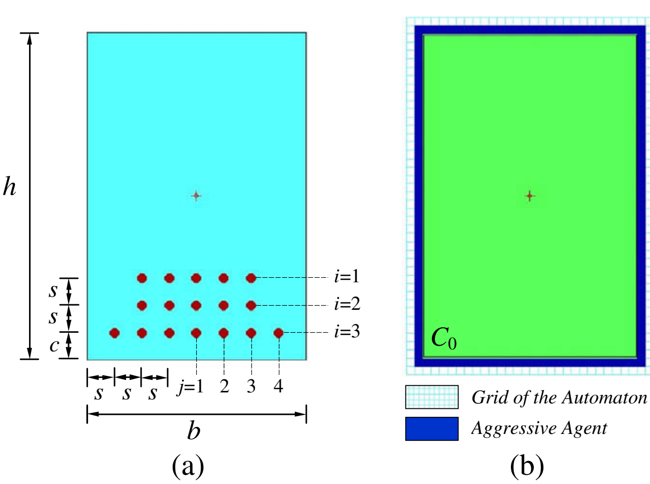


**Fig. 2.** Design model of the concrete cross-section. (a) Geometry of the cross-section and symmetric layout of the reinforcing bars $(i,j)$, $i = 1,\ldots,3$, $j = 1,\ldots,4$. (b) Grid of the cellular automaton and location of the aggressive agent.

$$c_{min} \leqslant c \leqslant c_{max} \tag{21}$$

$$\emptyset_i = 0 \quad \forall \emptyset_i < \emptyset_{min}, \quad i = 1,2 \tag{22}$$

$$\emptyset_i \leqslant \emptyset_{max}, \quad i = 1,\dots,3 \tag{23}$$

$$\emptyset_3 \geqslant \emptyset_{min} \tag{24}$$

$$\emptyset_{34} \geqslant \emptyset_{min} \tag{25}$$

where $h^*_{min} = 550$ mm, $h^*_{max} = 750$ mm, $c_{min} = 50$ mm, $c_{max} = 100$ mm, $\emptyset_{min} = 12$ mm, and $\emptyset_{max} = 26$ mm, with a discrete step size $\Delta\emptyset = 2$ mm between subsequent feasible diameters.

### 4.2. Diffusion process and corrosion damage

The aggressive agent is assumed to be located along the external boundary of the cross-section with prescribed concentration $C_0$, as shown in Fig. 2b. The diffusion process is studied by using the proposed cellular automata approach [14]. With reference to a diffusivity $D = 9.5 \times 10^{-12}$ m$^2$/s, the cellular automaton is defined by a grid dimension $\Delta z = 15.5$ mm and a time step $\Delta t = 0.1$ years. To highlight the role of stochastic effects in the mass transfer, the results obtained by assuming the evolutionary coefficient $\phi_0$ as deterministic or as random variable with symmetric triangular distribution in the range [0;1], are compared. With reference to a design solution with $h^* = 650$ and $c = 75$ mm, the time evolution of the diffusion process over a lifetime $T = 50$ years is described by the maps of concentration $C(\mathbf{z}, t)/C_0$ shown in Fig. 3 for deterministic mass transfer (Fig. 3a), and stochastic mass transfer (Fig. 3b).

The mechanical damage induced by diffusion is evaluated by assuming $C_{cr} = 0$, $C_s = C_0$, and $\Delta t_s = 50$ years. This damage model reproduces a deterioration process with severe corrosion of steel, as may occur for carbonated or heavily chloride-contaminated concrete and high relative humidity, conditions under which the corrosion rate can reach values above 100 μm/year [19]. Fig. 4 shows the time evolution of the damage indices of the steel bars associated to the diffusion process shown in Fig. 3. It is worth noting that for the case investigated in this study both deterministic and stochastic mass diffusion lead to comparable values of the damage indices over the lifetime $T$, with a maximum difference $\Delta\delta_{s,\max} < 0.03$.

### 4.3. Time-variant reliability and its threshold

The time-variant reliability index $\beta = \beta(t)$ is evaluated over a lifetime $T = 50$ years by assuming the material strengths as random variables. The compression strength $f_c$ is modeled by a lognormal distribution with mean value 35 MPa and standard deviation 5 MPa. The steel strength $f_y$ is modeled by a lognormal distribution with mean value 500 MPa and standard deviation 30 MPa. The time-variant resistant bending moment $M_R$ is computed by assuming for concrete in compression a stress block depth $0.8y$ over the neutral axis depth $y$, and an ultimate strain in compression $\varepsilon_{cu} = 0.35\%$. A time-invariant acting bending moment $M_A$ with lognormal distribution defined by a mean value $\mu_A = 300$ kN m and a coefficient of variation $\delta_A = 0.15$ is prescribed. Finally, by assuming that the frequency of periodic inspections or monitoring activities

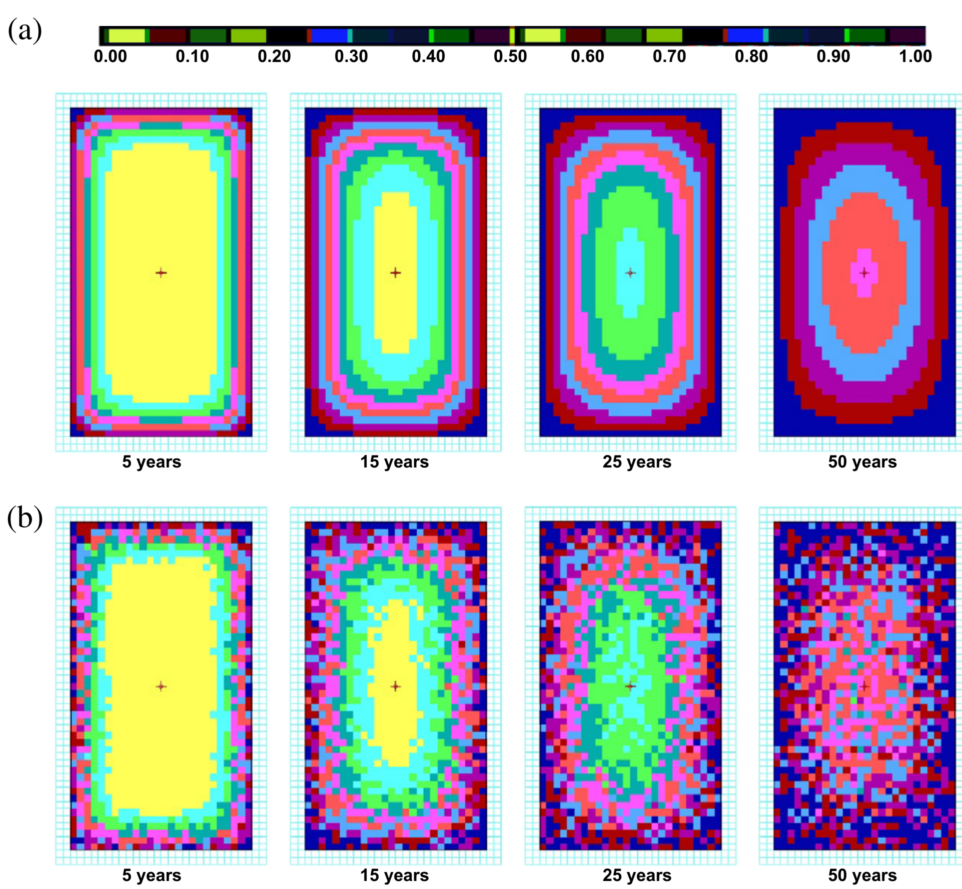


**Fig. 3.** Maps of concentration $C(\mathbf{z}, t)/C_0$ of the aggressive agent after 5, 15, 25, and 50 years from the initial time of diffusion penetration. (a) Deterministic diffusion. (b) Stochastic diffusion.

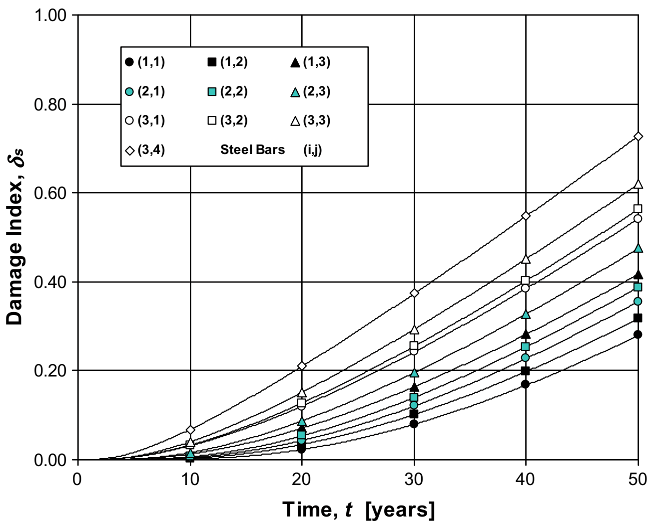


**Fig. 4.** Time evolution of the damage indices $\delta_s$ of the steel bars. The symbol $(i,j)$ refers to the steel bars of the design model shown in Fig. 2a.

will be progressively increased over the years to provide damage detection and early warning of failure, the following bilinear function $\bar{\beta} = \bar{\beta}(t)$ is adopted as time-variant reliability threshold:

$$\bar{\beta}(t) = \begin{cases} \bar{\beta}_0 + (\bar{\beta}_1 - \bar{\beta}_0)\frac{t}{t_1}, & 0 \leqslant t < t_1 \\ \bar{\beta}_1 + (\bar{\beta}_T - \bar{\beta}_1)\frac{t-t_1}{T-t_1}, & t_1 \leqslant t \leqslant T \end{cases} \quad (26)$$

with $\bar{\beta}_0 = 4.0$, $\bar{\beta}_1 = 3.5$, $\bar{\beta}_T = 1.5$, and $t_1 = 30$ years.

### 4.4. Discussion of the results

The minimum cost design of the RC cross-section depends on the unit cost ratio $\chi$. The role of this key parameter is investigated by solving the lifetime optimization problem for three different cost scenarios with $\chi = 20$, $\chi = 50$, and $\chi = 100$. A Monte Carlo simulation is performed on a sample of about 2000 time-variant cross-sectional analysis at each step of the solution process. The sample size has been chosen so to achieve a stable estimation of the time-variant statistical parameters of the safety factor. To highlight the role played by a lifetime approach to structural optimization, Fig. 5 and Table 1 make a comparison between the optimal solutions obtained respectively with a time-invariant formulation which considers the initial undamaged state only (Fig. 5a, Table 1a), and the proposed lifetime formulation where the time evolution of the structural performance is taken into account (Fig. 5b, Table 1b).

With respect to the time-invariant formulation, the lifetime approach leads to an increase of total cost by 16%, 14%, and 12%, for the unit cost ratio $\chi = 20$, $\chi = 50$, and $\chi = 100$, respectively. In general, as expected, the optimal depth $h^*$ of the cross-section

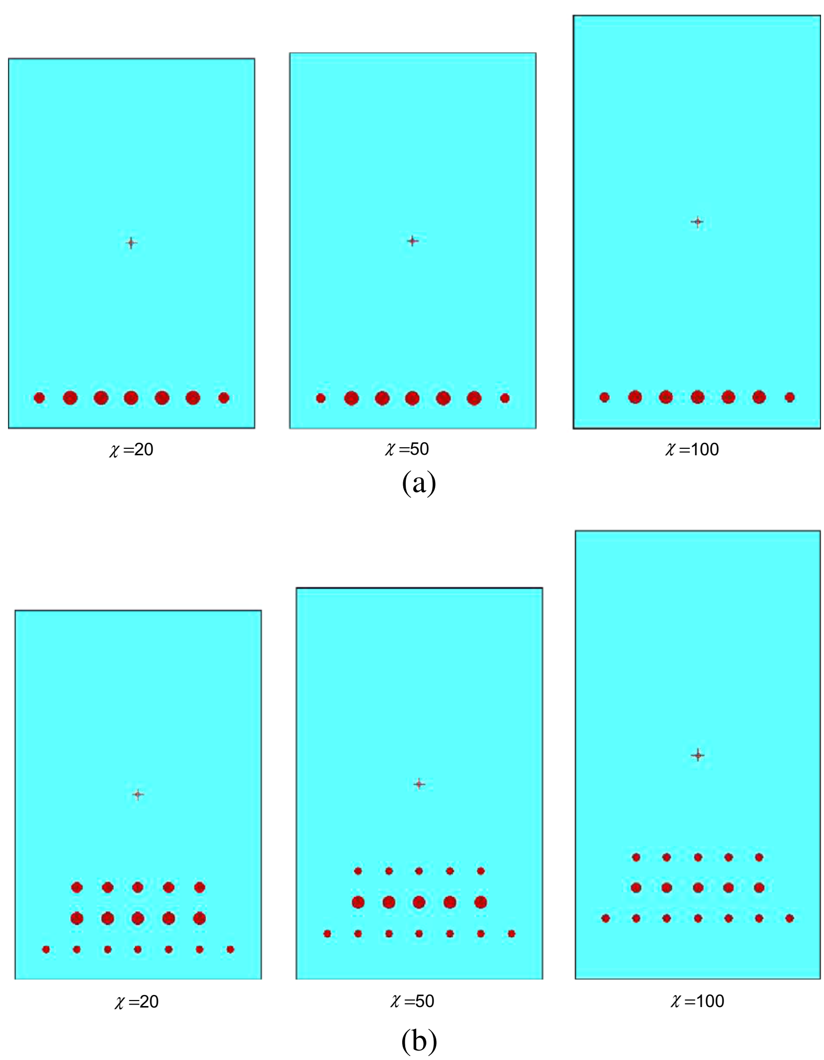


**Fig. 5.** Optimal design solutions (see also Table 1). (a) Time-invariant approach which considers the initial undamaged state only. (b) Lifetime approach where the time evolution of the structural performance is taken into account.

**Table 1**
Optimal design solutions (see also Fig. 5). (a) Time-invariant approach which considers the initial undamaged state only. (b) Lifetime approach where the time evolution of the structural performance is taken into account.

| $\chi$ | $h$ [mm] | $h^*$ [mm] | $c$ [mm] | $\emptyset_1$ [mm] | $\emptyset_2$ [mm] | $\emptyset_3$ [mm] | $\emptyset_{34}$ [mm] |
|---|---|---|---|---|---|---|---|
| *(a)* | | | | | | | |
| 20 | 600 | 550 | 50 | – | – | 22 | 16 |
| 50 | 610 | 560 | 50 | – | – | 22 | 14 |
| 100 | 665 | 615 | 50 | – | – | 20 | 16 |
| *(b)* | | | | | | | |
| 20 | 600 | 550 | 50 | 16 | 20 | 12 | 12 |
| 50 | 630 | 555 | 75 | 12 | 20 | 12 | 12 |
| 100 | 735 | 635 | 100 | 12 | 16 | 12 | 12 |

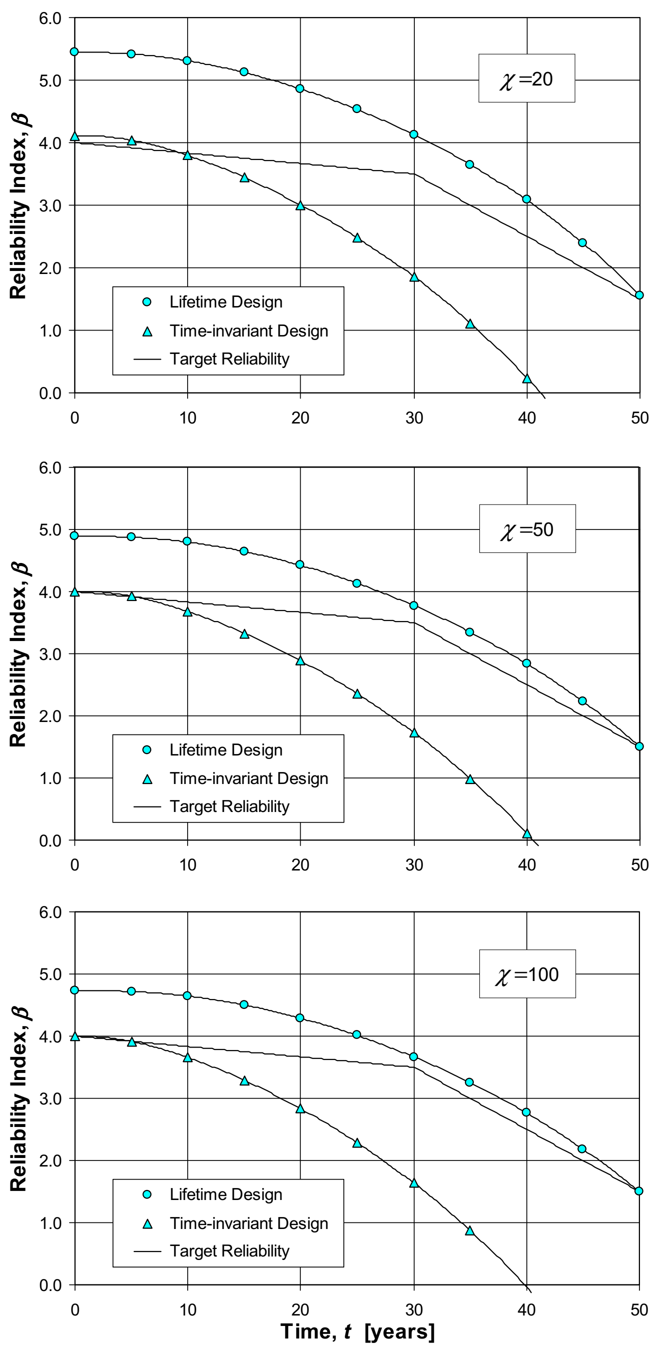


**Fig. 6.** Time evolution of the reliability index for the optimal design solutions shown in Fig. 5.

increases with the increasing of the unit cost ratio $\chi$. However, its value is not significantly affected by the type of time-invariant or lifetime formulation. The optimal values of the other design variables are instead strongly related to the effects of damage. In fact, if damage is not considered, the minimum feasible steel area of reinforcement is associated with the maximum depth of the steel bars (i.e. the layer $i = 3$), and the minimum concrete cover is selected since its value does not affect the resistant bending moment (Fig. 5a). On the contrary, if the lifetime structural performance is considered, the location of each layer of steel bars, as well as the value of the concrete cover, can play a crucial role in the definition of the minimum feasible area of reinforcement. In fact, due to the damage process induced by diffusion, a suitable balance between the opposite trend in maximizing the bars depth and minimizing the damage effects is required. The way to achieve such trade-off depends on the cost scenario, since an increasing in the unit cost ratio $\chi$ involves not only a reduction of the total area of steel reinforcement, but also an improvement of the reinforcement protection from the aggressive agent, that is obtained with higher depth values for the steel bars with larger diameters and larger values of the concrete cover (Fig. 5b). These results highlight that the reinforcement layout and the concrete cover may significantly affect the time evolution of structural performance, and they should be considered as key factors for a lifetime optimum design.

Finally, Fig. 6 shows the time evolution of the reliability index for the optimal solutions shown in Fig. 5. It can be noted that a lifetime optimization approach is required to satisfy the reliability constraint $\beta(t) \geqslant \bar{\beta}(t)$ not only at the initial time, but over the prescribed lifetime $T$.

## 5. Conclusions

A lifetime approach to reliability-based optimization of RC cross-sections subjected to diffusive attacks from environmental aggressive agents has been presented. The lifetime probabilistic optimization has been formulated in a restrictive case in which the optimization consists of the minimization of the cost of a time-variant constraint on the structural reliability without considering maintenance. The role played by a lifetime approach to structural optimization has been shown by comparing the optimal solutions obtained with a classical time-invariant formulation, which considers the initial undamaged state only, and the proposed lifetime formulation, where the time evolution of the structural performance is taken into account. The obtained results showed that in a lifetime-oriented design the minimum feasible area of reinforcement is not associated with the maximum depth of the steel bars over the concrete cross-section, as expected in a classical time-invariant approach. In fact, the amount and location of the steel reinforcement and the value of the concrete cover play a crucial role in the achievement of the desired lifetime optimal performance.

To emphasize the importance of a lifetime approach in structural optimization, the present study focused on the minimum cost design of RC cross-sections with respect to the ultimate limit state under the assumptions of steel corrosion and randomness of the material strengths only. Further studies are required to investigate the effects of maintenance interventions on the optimum structural design along the lines proposed in [3,7], as well as the role of other objectives, such as lifetime cost, and design variables, including cross-sectional shape and bar spacing, design constraints related to serviceability limit states, additional damage mechanisms, such as concrete deterioration, and other sources of uncertainty, including randomness of geometrical parameters, diffusive scenario, and damage rates. Also, research on determination of optimal target reliabilities for design and upgrading of concrete structures along the lines proposed in [30] is necessary. Finally, recent studies in the field of multi-objective optimization of deteriorating structures are useful in supporting decisions when cost and performance are in conflict [8,10,13].

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