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Morphologic Evolutionary Structural Optimization

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1. Abstract

This paper presents a new approach to evolutionary optimization of structural morphology based on two evolutionary paradigms. In the first one, the structural shape is optimized by simulating the growth process of natural structures. In the second one, the structural topology is optimized by removing material at low stress level. The proposed approach, called Morphologic Evolutionary Structural Optimization (MESO), combines these two design paradigms and leads to optimize simultaneously structural shape and structural topology. The extension of the proposed procedure to evolutionary optimization of non-homogeneous structures is also presented. The effectiveness of the procedure is shown through applications.

2. Keywords: Topology optimization; Shape optimization; Evolutionary procedures; Non-homogeneous structures.

3. Introduction

During the last two decades, several approaches to structural optimization have been developed by using evolutionary procedures which operate on the basis of some analogies with the growing and the evolutionary processes of natural systems. Such methods are based on the simple concept that by slowly removing and/or reshaping regions of inefficient material, belonging to a given over-designed structure, its shape and topology evolve toward an optimum configuration. Among others, the attention of researchers focused on two main approaches. In the first one, the structural shape is modified by simulating the Biological Growth (BG) of natural structures like bones and trees [5], [6]. In the second one, known as Evolutionary Structural Optimization (ESO), the structural topology is optimized by removing material at low stress level [7], [4]. BG and ESO procedures have been proven to be very effective when applied to shape optimization or topology optimization, respectively [2]. However, due to their nature, they are less suitable to solve problems where optimal shape and optimal topology are contemporaneously searched for.

Based on such considerations, this paper presents a new approach to evolutionary optimization of structural morphology developed by combining the main aspects of both BG and ESO procedures. In this way the new approach, called Morphologic Evolutionary Structural Optimization (MESO), allows to optimize simultaneously structural shape and structural topology [1]. The MESO procedure has been originally developed for homogeneous structures. However, many structures exhibit low strength in tension, like those made of stone or concrete, or in compression, like those subjected to buckling phenomena. To account for such cases, in which the optimal structural morphology should be defined by limiting the amount of material subjected to critical stress states, the concept of tension and compression dominated material is introduced. Based on this concept, the actual design domain is subdivided at each step in two or more parts, and in each of them a different material is considered. In this way, the MESO procedure is extended to non-homogeneous structures composed by several materials. The effectiveness of the proposed procedure is shown through applications.

4. BG and ESO Procedures

4.1. Biological Growth

Natural structures are known to evolve by adapting themselves to the applied loads according to the axiom of uniform stress, which states that in the optimal configuration the stress field distribution tends to be fairly regular over the structure. BG procedure operates according to the axiom of uniform stress by gradually modifying the structural shape in such a way that material is added in the zones with high stress concentrations and removed from under-loaded zones [5], [6]. The numerical simulation of the growth mechanism is obtained through three steps.

(1) Basic Step. A finite element analysis is performed to obtain the stress distribution over the structure.
(2) Swelling Step. A strain distribution is computed in proportion to the deviation of a reference stress parameter (for example the von Mises stress) from its mean value computed over the whole structural volume. This strain distribution is applied to the structure and the corresponding swelling displacements are evaluated. During this step additional geometrical design constraints can be taken into account by replacing the actual boundary conditions of the swelling model in such a way that swelling displacements which violate the constraints are not allowed [2].
(3) Update Step. The location of each node of the finite element model at current generation is updated in proportion to the swelling displacements obtained at the previous swelling step. The parameter of proportionality may be either fixed at the first generation and then considered time-independent, or varied during the evolution. In any case, its value should be chosen to assure noticeable shape variations Δx and progressively decreasing driving forces [2].

4.2. Evolutionary Structural Optimization

ESO procedures are based on the simple concept that by slowly removing regions of inefficient material, belonging to a given over-designed structure, its topology evolves toward an optimum configuration [7], [4]. The initial domain is subdivided in finite elements and a structural analysis is carried out. A representative quantity of the structural response, usually identified with the von Mises stress, is then evaluated at the element level and compared with a prescribed percentage *RR* (rejection ratio) of a reference value, for instance the maximum value computed over the whole structure. If such threshold is not reached, the material inside the corresponding elements is considered to be inefficient and it is removed by degrading its constitutive properties, typically the Young modulus. The rejection ratio *RR* is progressively increased during the iterative procedure by means of an evolutionary rate *ER* to assure a gradual evolution of the process.

5. MESO Procedure

5. 1. Basic Steps

The MESO procedure combines the main aspects of both BG and ESO procedures. The basic steps of this evolutionary procedure are briefly recalled in the following. A more detailed description can be found in [1].

Step 1: *Topology optimization with removal of the inefficient material along the free boundary.*
A finite element model of an initial over-dimensioned design domain is developed. An evolutionary optimization of the structural topology is then performed based on the ESO procedure applied only to the finite elements along the free boundary of the design domain. During the first phase of this process a minimum percentage of material volume $V_{\min}$ is gradually removed. Subsequently, new iterations are performed until a prescribed percentage of material volume $V_{\max}$ is removed, or until an internal element is verified to be less efficient than the elements along the boundary. In fact, in this last case the internal structural topology needs to be updated, for example by introducing a new cavity (see Step 5).

Step 2: *Update of the finite element model for subsequent shape optimization.*
At the end of the topology optimization phase the finite element model is usually characterized by an irregular boundary. To avoid numerical problems and to improve the computational effectiveness of the subsequent BG shape optimization process, this model needs to be updated by smoothing the irregular boundary. This is achieved by introducing a set of control points along the boundary and by remeshing the finite element model with reference to these points. The control points must be chosen to respect the presence of applied loads, imposed displacements, and to maintain the eventual symmetry of the system. It is clear that by increasing the number of control points we increase the accuracy of the remeshing process in reproducing the model obtained from the previous topology optimization step, but, at the same time, we also decrease the effectiveness of both the smoothing procedure and the subsequent shape optimization process. As a general criterion, a minimum distance $d_{\min}$ between two subsequent control points can be selected as a multiple *m* of the characteristic size *s* of the finite elements:

$$d_{\min} = m \cdot s \tag{1}$$

Figure 1 shows the effects of the parameter *m* in the smoothing process.

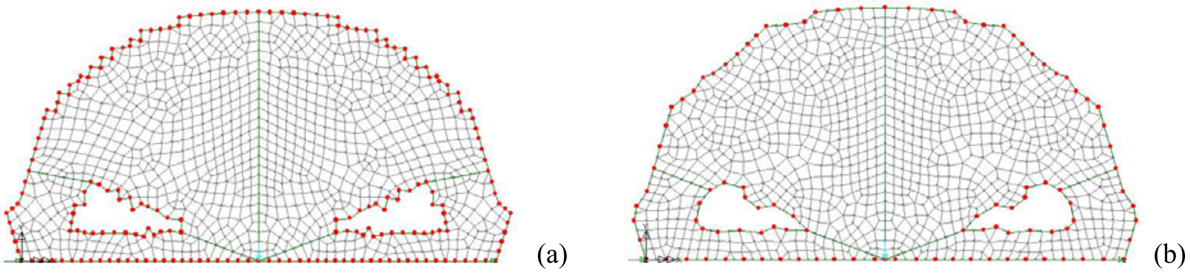


Figure 1. Choice of the control points. Smoothing of the finite element model with (a) *m*=1, and (b) *m*=2.

Step 3: *Shape optimization with moving of the external free boundary.*
A shape optimization is performed on the finite element model obtained from step 2 by using a BG procedure. During the first phase of this process a minimum number of iterations $N_{\min}$ are performed. Subsequently, the process is repeated until a prescribed number $N_{\max}$ of iterations is reached, or until the overall structural efficiency can no longer be improved. The structural efficiency I_i at current iteration *i* is measured by means of the following dimensionless index:

$$I_i = \frac{V_i \bar{\sigma}_i}{V_{0,i} \bar{\sigma}_{0,i}} \tag{2}$$

where $\bar{\sigma}$ is the average stress computed over the material volume *V*, and the subscript “0” refers to the initial structure.

To improve the effectiveness of this process the model update required at each iteration is performed by moving the control points only, and by remeshing the finite element model with reference to these points.

Step 4: *Update of the finite element model for subsequent topology optimization*.
At the end of the shape optimization phase the finite element model is usually characterized by an irregular discretization. To avoid numerical problems and to improve the effectiveness of the subsequent ESO topology optimization process, the finite elements of the model are grouped in fairly regular elimination units, as shown in Figure 2. In this way, the ESO procedure operates by removing not single finite elements, but groups of them associated with elimination units.

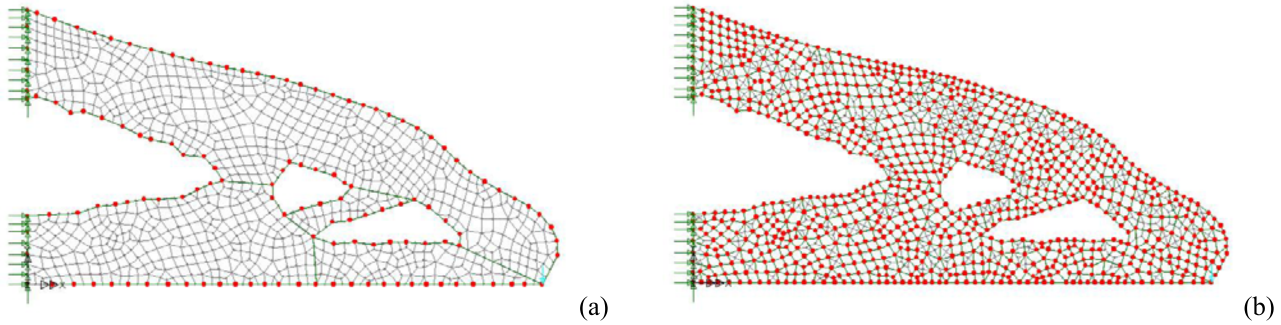


Figure 2. Generation of the elimination units. (a) Finite element model at the end of the shape optimization step, and (b) subdivision in elimination units for the topology optimization step.

Step 5: *Creation of a new internal cavity*.
At the end of the shape optimization process the structural efficiency of each element of the model is verified. In case there is an internal element less efficient than the elements located along the free boundary of the model, a new internal cavity is created by eliminating this element, or the whole elimination unit to which the element belongs.

5.2. Evolutionary Design of Bridge Structure
The MESO procedure is applied to the optimization of the structural morphology of a single span bridge structure. The position of both the deck and the supports is assumed to be fixed. Figure 3 shows the geometric proportions of the initial design domain and the load condition along the deck. Two cases are considered. In the first one, the deck is located at the top of the design domain (Figure 3.a), in the second case it is located at the bottom of the design domain (Figure 3.b).

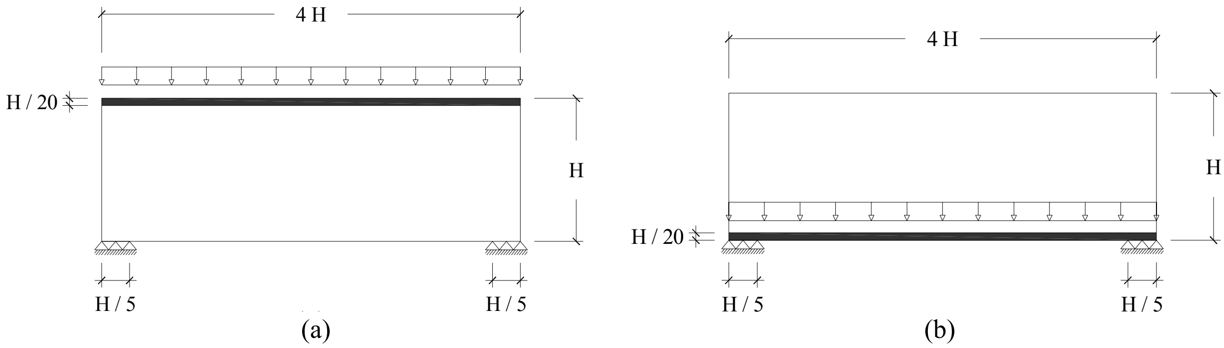


Figure 3. Optimization of a single span bridge structure. Geometry and boundary conditions of the initial design domain: bridge deck located (a) at the top and (b) at the bottom of the design domain.

Young modulus and Poisson coefficient are $E = 100$GPa and $\upsilon = 0.3$, respectively. The initial design domain is discretized by using finite elements with mesh size varying between 0.03H and 0.06H. The main parameters of MESO procedure used in the evolutionary optimization process are: $ER = 1\%$, $RR = 1\%$, $V_{\min} = 10\%$, $V_{\max} = 15\%$, $\Delta x = H/20$, $N_{\min} = 10$. The swelling models adopted during the shape optimization are developed by assuming a fixed constraint for each point of the deck, which is enforced in this way to maintain its original initial position during the optimization process.
Figures 4 and 5 show the configurations obtained during the optimization process for case (a) and case (b), respectively. The contoured maps refer to von Mises stress distribution. The obtained results prove the effectiveness of the proposed procedure, which is able to generate optimized structures with regular shape and quite uniform stress distributions, and highlight the positive role played by the synergetic cooperation between BG and ESO procedures, which allows to fully exploit the characteristics of the evolutionary approach.

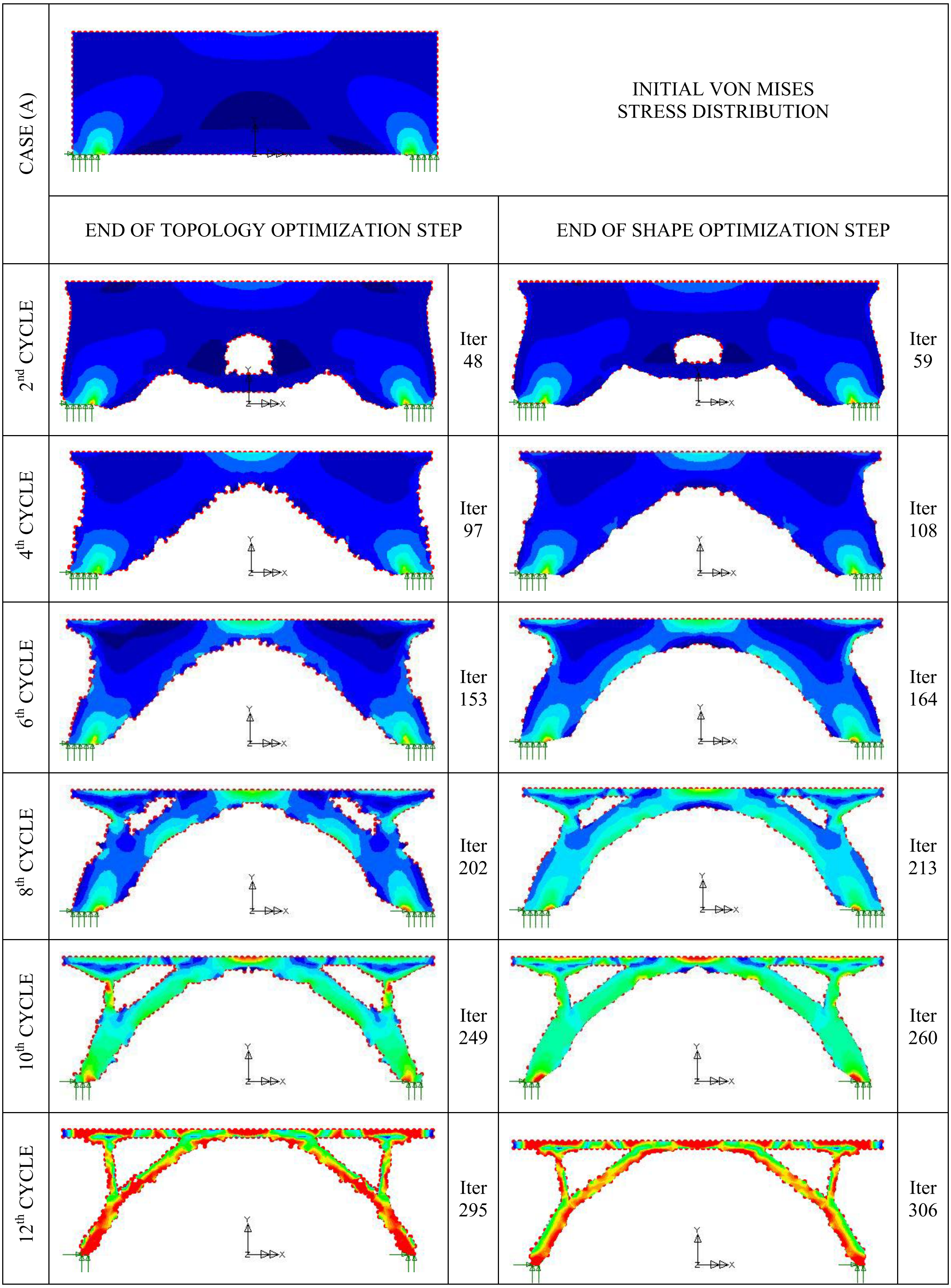


Figure 4. Optimization of a single span bridge structure. Bridge deck located at the top of the design domain.

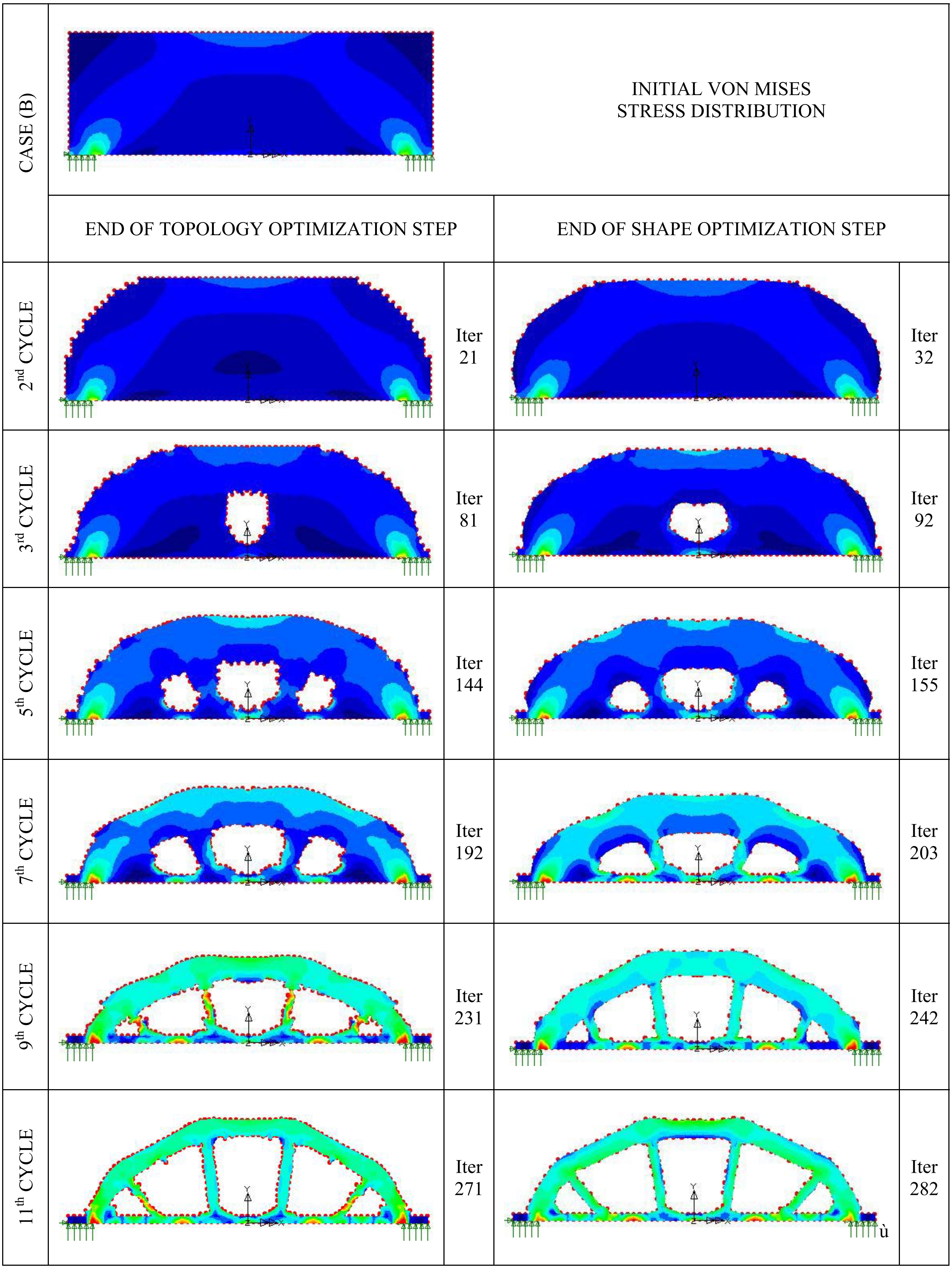


Figure 5. Optimization of a single span bridge structure. Bridge deck located at the bottom of the design domain.

6. MESO for Non-homogeneous Structures

6.1. Tension and Compression Dominated Materials

The MESO procedure has been originally developed for structures made of homogeneous materials having symmetric behavior in tension and compression. However, many structures exhibit low strength in tension, like those made of stone or concrete, or in compression, like those subjected to buckling phenomena. To account for such cases, in which the optimal structural morphology should be defined by limiting the amount of material subjected to critical stress states, the concept of tension and compression dominated material has been introduced [3], [2].

As shown in Figure 6, material is considered tension (compression) dominated if the maximum (minimum) principal stress is of tension (compression) type. Based on this concept, the actual domain Ω is subdivided at each step into two parts, tension dominated Ω_T and compression dominated Ω_C, and in each of them the material efficiency is verified by using the absolute values of the principal stresses instead of the von Mises stresses. Specifically, for materials having low tensile (compressive) strength the efficiency is verified only in the tension (compression) dominated volume Ω_T (Ω_C) with reference to the minimum principal stress σ_2 (maximum principal stress σ_1).

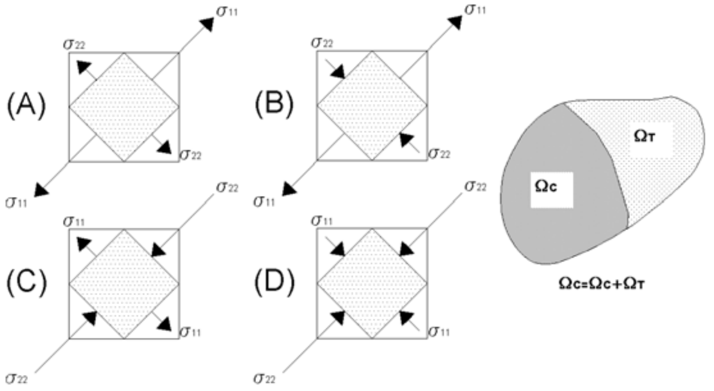


Figure 6. Tension (A, B, Ω_T) and compression (C, D, Ω_C) dominated material.

6.2. Static and Dynamic Evolutionary Approach

Based on the concept of tension and compression dominated material, the MESO procedure can be easily specialized to find structures whose predominant stress is of a prescribed type. However, a consistent structural design should prefer the use of different materials, each one able to withstand a different type of stress state, rather than a less rational modification of the whole structural morphology aimed to cover a specific material weakness. A possible way to extend the proposed approach to non-homogeneous structures is to subdivide the initial domain into several parts, and by assigning to each part a different material. This leads to a *static* evolutionary approach, where each prescribed part evolves according to a different criterion. Some basic applications of a static ESO approach can be found in [2]. In this paper, such concepts are implemented in the MESO procedure by adopting a *dynamic* evolutionary approach, where also the subdivision of the design domain in different materials is updated during the optimization process. The applications presented in the following refer to structures composed by two materials, but the approach is general and can be easily applied to the case of multiple materials.

6.3. Evolutionary Design of Bridge Structure with a Passageway

The MESO procedure is here applied to the optimization of the structural morphology of a three-span bridge structure [2]. The position of both the deck and the supports is assumed to be fixed and a free space for navigation is provided under the deck. Figure 7 shows the geometric proportions and the design domain. The deck is assumed to be uniformly loaded.

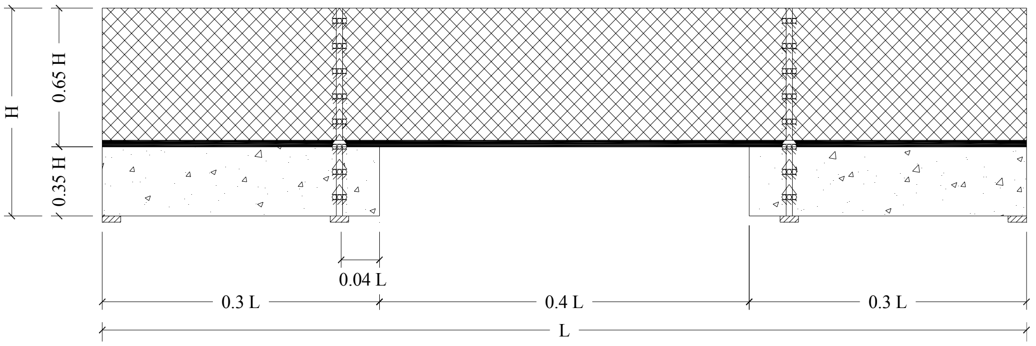
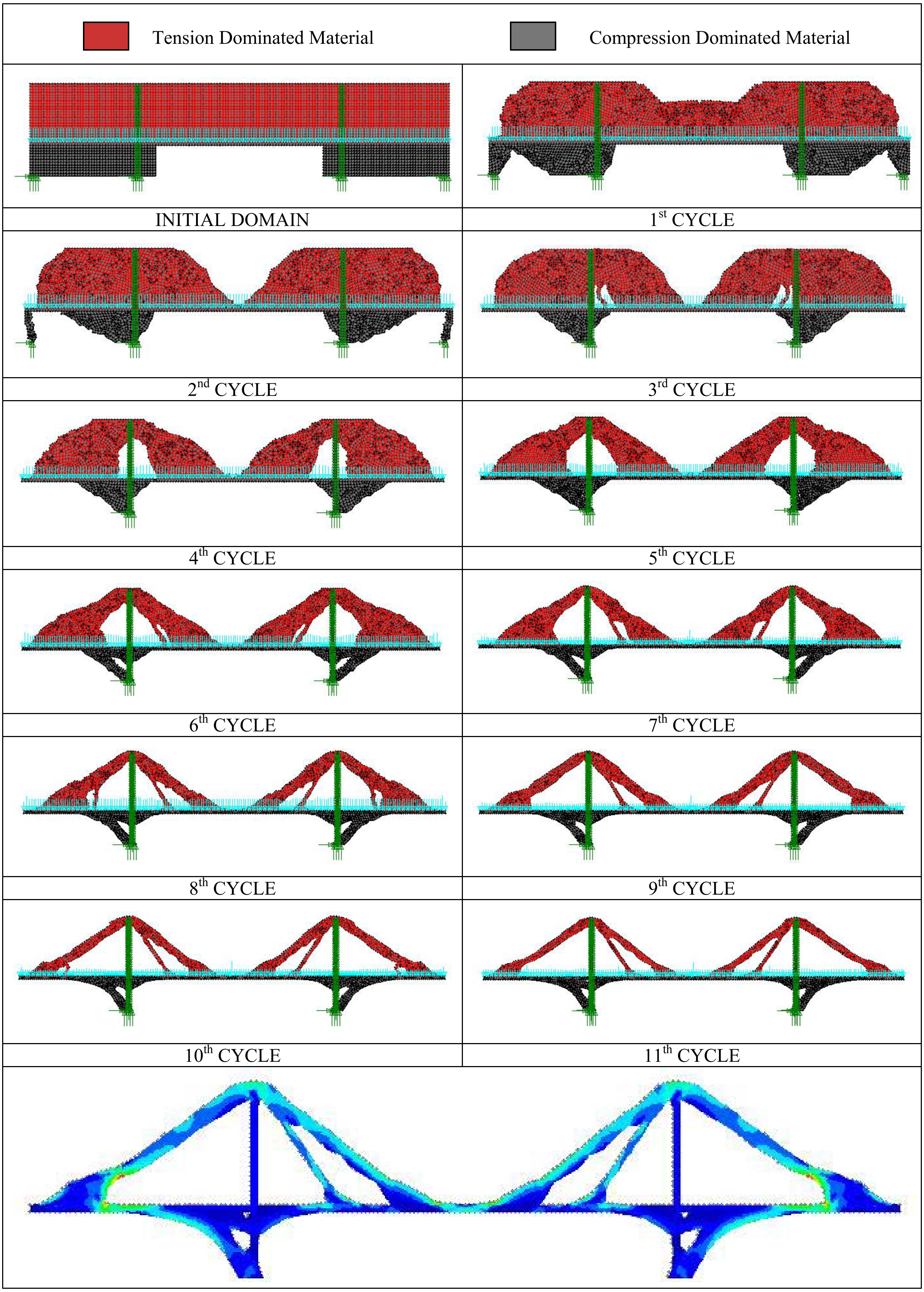


Figure 7. Optimization of a three-span span non homogeneous bridge structure. Design domain.



INITIAL DOMAIN

1st CYCLE

2nd CYCLE

3rd CYCLE

4th CYCLE

5th CYCLE

6th CYCLE

7th CYCLE

8th CYCLE

9th CYCLE

10th CYCLE

11th CYCLE

Figure 8. Optimization of a three-span span non homogeneous bridge structure. Static MESO approach.

INITIAL DOMAIN

1st CYCLE

2nd CYCLE

3rd CYCLE

4th CYCLE

5th CYCLE

6th CYCLE

7th CYCLE

8th CYCLE

9th CYCLE

10th CYCLE

11th CYCLE

Figure 9. Optimization of a three-span span non homogeneous bridge structure. Dynamic MESO approach.

The design domain is assumed as composed by two materials, the first one having low strength in tension with $E = 30$GPa and $\upsilon = 0.3$, the second one having low strength in compression with $E = 209$GPa and $\upsilon = 0.3$. The initial design domain is discretized by using finite elements with mesh size varying between 0.03H and 0.04H. The main parameters of MESO procedure used in the evolutionary optimization process are: $ER = 1\%$, $RR = 1\%$, $V_{\min} = 10\%$, $V_{\max} = 15\%$, $\Delta x = H/20$, $N_{\min} = 5$. The swelling models adopted during the shape optimization are developed by assuming a fixed constraint for each point of the deck, which is enforced in this way to maintain its original initial position during the optimization process.

Two cases are studied. In the first case, the design domain is subdivided into two parts with respect to the deck position. The material with low strength in compression is prescribed in the upper part, while the material with low strength in tension is prescribed in the lower part. To allow the activation of a cable-stayed scheme, two axially rigid pylons are also added to the design domain. Figure 8 shows the results of a static MESO evolutionary optimization, which confirm the expected primacy of the cable-stayed solution for the adopted material distribution. However, the primacy of this scheme no longer holds in case a dynamic MESO evolutionary optimization is performed. In fact, the balanced arch scheme shown in Figure 9 is preferred if a redistribution of the material type is allowed during the optimization process.

7. Conclusions

A novel approach to evolutionary optimization of structural morphology has been presented. This approach, called Morphologic Evolutionary Structural Optimization (MESO), is based on two evolutionary paradigms. In the first one, the structural shape is optimized by simulating the growth process of natural structures. In the second one, the structural topology is optimized by removing material at low stress level. The proposed approach combines these two design paradigms and leads to optimize simultaneously structural shape and structural topology. The MESO procedure, originally developed for homogeneous structures, has been also extended to the case of non-homogeneous structures. The obtained application results proved the effectiveness of the proposed approach, and highlighted the positive role played by the synergetic cooperation between the two adopted evolutionary paradigms.

8. References

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