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Reliability of material and geometrically non-linear reinforced and prestressed concrete structures

Fabio Biondini [a,*], Franco Bontempi [b], Dan M. Frangopol [c], Pier Giorgio Malerba [a]

[a] *Department of Structural Engineering, Technical University of Milan, Piazza L. da Vinci, 32, Milan 20133, Italy*
[b] *Department of Structural and Geotechnical Engineering, University of Rome, "La Sapienza", Via Eudossiana, 18-00184 Rome, Italy*
[c] *Department of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder, CO 80309-0428, USA*

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Abstract

A numerical approach to the reliability analysis of reinforced and prestressed concrete structures is presented. The problem is formulated in terms of the probabilistic safety factor and the structural reliability is evaluated by Monte Carlo simulation. The cumulative distribution of the safety factor associated with each limit state is derived and a reliability index is evaluated. The proposed procedure is applied to reliability analysis of an existing prestressed concrete arch bridge.

Keywords: Concrete structures; Non-linear analysis; Structural reliability; Bridges; Simulation

1. Introduction

This paper considers a direct and systematic approach to the reliability analysis of reinforced and prestressed concrete structures subjected to static loads [4]. The structural reliability is evaluated by Monte Carlo simulation. Therefore, repeated non-linear analyses are carried out giving outcomes from a set of basic variables which define the structural problem (e.g. mechanical and geometrical properties, dead and live loads, prestressing forces, etc.). The results of the analysis associated to each singular realization are then statistically examined and used to evaluate the reliability index associated with each considered limit state. The proposed procedure is finally applied to the reliability assessment of an existing arch bridge. The structure is modeled by using a composite reinforced/prestressed concrete beam element, whose formulation accounts for the mechanical non-linearity due to the constitutive properties of materials (i.e. cracking, softening and crushing of concrete; yielding, hardening and failure of steel; prestressing action), as well as for the geometrical non-linearity due to second order effects.

2. Probability of failure and reliability index

A structure is safe if the applied actions S are less than its resistance R. The problem may also be formulated in terms of the probabilistic safety factor $\Theta = R/S$. Let θ be a particular outcome of the random variable Θ. The probability of failure can be evaluated by the integration of the density probability function $f_\Theta(\theta)$ within the failure domain $D = \{\theta | \theta < 1\}$:

$$P_F = P(\Theta < 1) = \int_D f_\Theta(\theta)\,d\theta. \qquad (1)$$

The above equation is often approximated as

$$P_F = \Phi(-\beta), \qquad (2)$$

* Corresponding author. Tel.: +39-02-2399-4394; fax: +39-02-2399-4220.
E-mail address: biondini@stru.polimi.it (F. Biondini).

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where Φ is the standard normal cumulative probability function and $\beta = -\Phi^{-1}(P_F)$ is the *reliability index* which represents, in the space of the standard normal variables (zero mean values and unit standard deviations), the shortest distance from the origin to the surface which defines the limit state.

3. Reliability assessment by simulation methods

In practice the density function $f_\Theta(\theta)$ is not known and at the most some information is available only about a set of n basic random variables $\mathbf{X} = [X_1 \quad X_2 \quad \cdots \quad X_n]^T$ which define the structural problem (e.g. mechanical and geometrical properties, dead and live loads, prestressing actions, etc.).

Moreover, in concrete design the limit states are usually formulated in terms of functions of random variables $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$ which describe the structural response (e.g. stresses, strains, etc.), and such derivation is generally only available in an implicit form. A numerical approach is then required and the reliability analysis can be performed by Monte Carlo simulation [6], where repeated analyses are carried out with random outcomes of the basic variables $\mathbf{X}$ generated in accordance to their marginal density functions $f_{X_i}(x_i)$, $i = 1, \ldots, n$. Based on the sample obtained through the simulation process, the density function $f_\Theta(\theta)$ or the cumulative function $F_\Theta(\theta)$ can be derived for each given limit state $h(\mathbf{Y}) = 0$, and the corresponding probability of failure $P_F = F_\Theta(1)$, as well as the reliability index $\beta = -\Phi^{-1}[F_\Theta(1)]$, can be evaluated.

An analytical interpolation of the numerical results can also be attempted, for example in terms of cumulative function $F_\Theta(\theta)$. To this aim, a fairly regular and non-decreasing function $F_\Theta(\theta)$ with

$$\lim_{\theta \to -\infty} F_\Theta(\theta) = 0, \quad \lim_{\theta \to +\infty} F_\Theta(\theta) = 1 \tag{3}$$

can be chosen as described in Biondini et al. [1]:

$$F_\Theta(\theta) = \frac{1}{2}\left[1 + \tanh\left(\sum_{k=0}^{K} c_k \theta^k\right)\right]. \tag{4}$$

A good accuracy is usually achieved by assuming $K = 5$ and the coefficients c_k are identified through a least square minimization.

4. Failure criteria for concrete structures

4.1. Serviceability limit states

Splitting cracks and considerable creep effects may occur if the compression stresses σ_c in concrete are too high. Besides, excessive stresses either in reinforcing steel σ_s or in prestressing steel σ_p can lead to unacceptable crack patterns. Excessive displacements $\mathbf{s}$ may also involve loss of serviceability and then have to be limited within assigned bounds $\mathbf{s}^-$ and $\mathbf{s}^+$. Based on these considerations, the following constraints account for adequate durability at the serviceability stage:

$$S1: -\sigma_c \leqslant -\alpha_c f_{ck}, \tag{5a}$$

$$S2: |\sigma_s| \leqslant \alpha_s f_{syk}, \tag{5b}$$

$$S3: |\sigma_p| \leqslant \alpha_p f_{pyk}, \tag{5c}$$

$$S4: \mathbf{s}^- \leqslant \mathbf{s} \leqslant \mathbf{s}^+, \tag{5d}$$

where α_c, α_s and α_p are reduction factors of the characteristic values f_{ck}, f_{syk}, and f_{pyk} of the material strengths.

4.2. Ultimate limit states

When the strain in concrete ε_c, or in the reinforcing steel ε_s, or in the prestressing steel ε_p reaches a limit value ε_{cu}, ε_{su} or ε_{pu}, respectively, the failure of the corresponding cross-section occurs. However, the failure of a single cross-section does not necessarily lead to the failure of the whole structure, since the latter is caused by the loss of equilibrium arising when the reactions $\mathbf{r}$ requested for the loads $\mathbf{f}$ can no longer be developed. Therefore, the following ultimate conditions have to be verified:

$$U1: -\varepsilon_c \leqslant -\varepsilon_{cu}, \tag{6}$$

$$U2: |\varepsilon_s| \leqslant \varepsilon_{su}, \tag{7}$$

$$U3: |\varepsilon_p| \varepsilon_{pu}, \tag{8}$$

$$U4: \mathbf{f} \leqslant \mathbf{r}. \tag{9}$$

4.3. Probabilistic safety factors and limit load multipliers

Since these limit states refer to internal quantities of the system, a check of the structural performance through a non-linear analysis needs to be carried out at the load level. To this aim, it is useful to assume $\mathbf{f} = \mathbf{g} + \Theta\mathbf{q}$, where $\mathbf{g}$ is a vector of dead loads and prestressing actions, and $\mathbf{q}$ is a vector of live loads whose intensity varies proportionally to a unique multiplier $\Theta \geqslant 0$. Using these vectors, the serviceability and ultimate limit states previously defined can be directly described in terms of the corresponding limit load multipliers Θ, which assume the role of probabilistic safety factors.

It is worth noting that non-linear analysis plays a fundamental role in the evaluation of the limit load multipliers. In fact, for reinforced and prestressed concrete structures the distribution of stresses and strains in the materials (concrete, reinforcing and prestressing steel), as well as the magnitude of the displacements and the collapse loads, depend on non-linear phenomena as cracking and crushing of the concrete matrix, yielding of the reinforcement bars and/or of the prestressing cables, second order geometrical effects, etc. As a consequence, the investigated ultimate limit states cannot be investigated in the linear field and, in most cases, such kind of structures should be analyzed by taking material and, possibly, geometrical non-linearity into account if realistic results under all load levels are needed.

Nowadays, non-linear analysis is a tool that can be applied more easily than in the past. In many reports and normative codes this aspect is recognized and it is highlighted that non-linear analysis can give more meaningful results than linear analysis. For these reasons, a new trend in design is spreading, where the usual procedure of non-linear verification of cross-sections on the basis of the results of linear analyses tends to be replaced by a full non-linear analysis where the structural safety is evaluated at the load level.

5. Application to an existing arch bridge

The proposed procedure is now applied to the reliability analysis of the existing three hinge arch bridge shown in Fig. 1 [7]. The total length of the bridge is 158 m, with a central span of 125 m, and the total width of the deck 8.10 m (Fig. 2). The box-girder cross-section has the width 5.00 m and height varying from 7.00 m at the abutments to 2.20 m at the crown (Fig. 3). The layout of the prestressing cables is shown in Fig. 4. The nominal value of the prestressing stress is $\sigma_{p,\mathrm{nom}} = 1200$ MPa. The number of reinforcement bars varies from a minimum of $108\varnothing 22$ at the crown to a maximum $164\varnothing 22$ at the abutments. The bridge was built by using prestressed lightweight concrete with the following material properties:

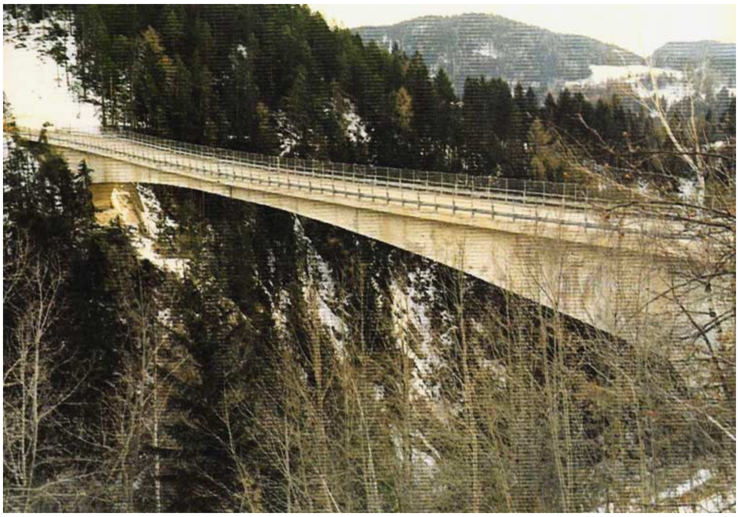


Fig. 1. View of the arch bridge over the Rio Avelengo, Bolzano, Italy (reprinted with permission from *L'industria Italiana del Cemento*—[7]).

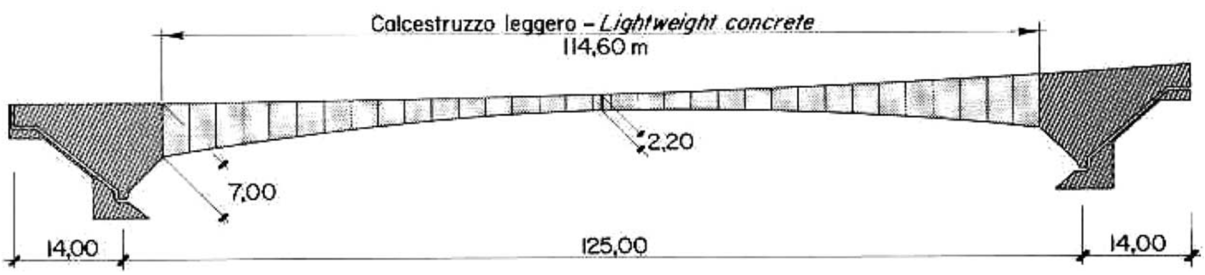


Fig. 2. Schematic view and main geometrical dimensions of the bridge (reprinted with permission from *L'industria Italiana del Cemento*—[7]).

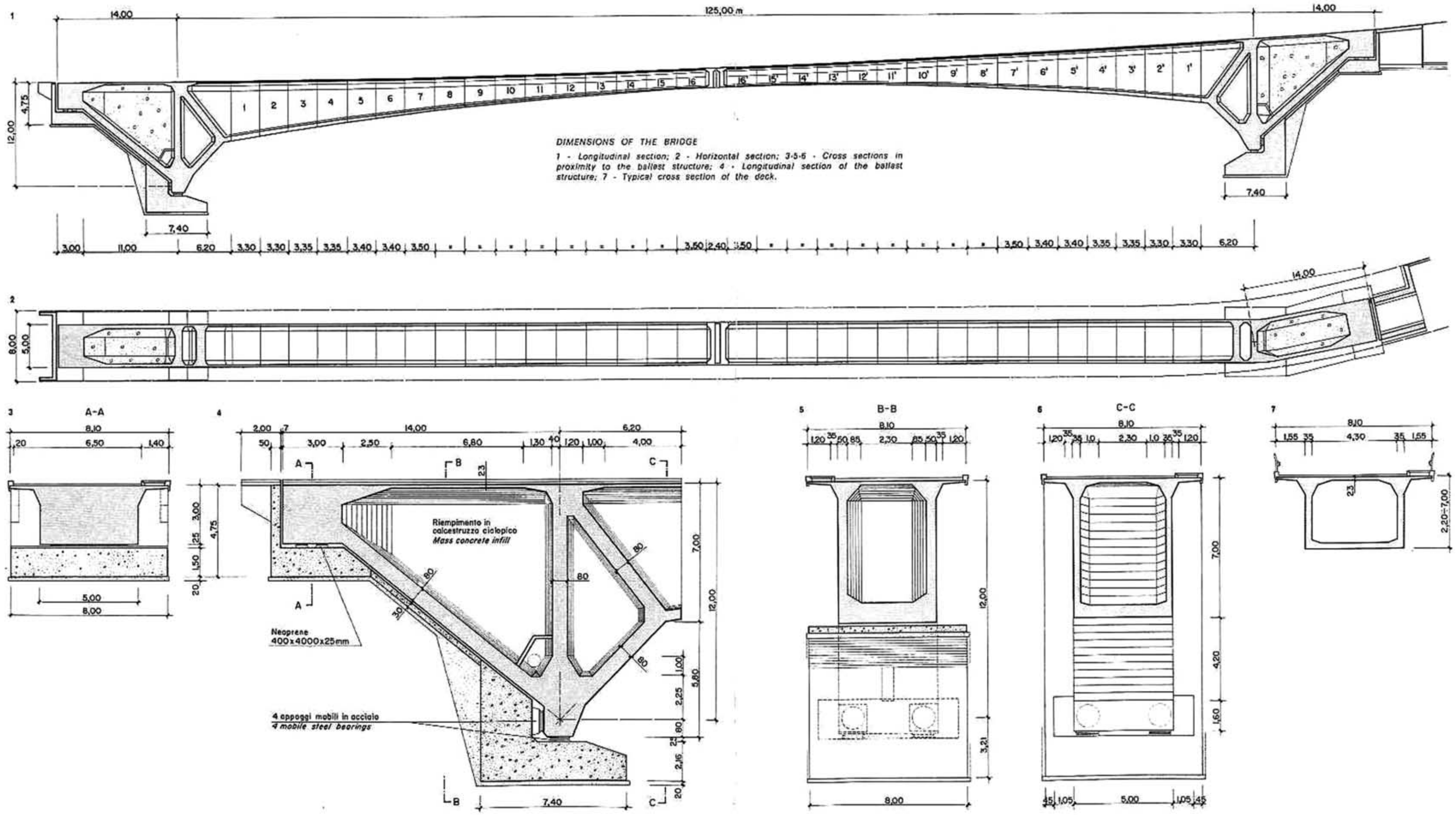


Fig. 3. Longitudinal, horizontal and transversal cross-sectional views of the bridge (reprinted with permission from *L'industria Italiana del Cemento*—[7]).

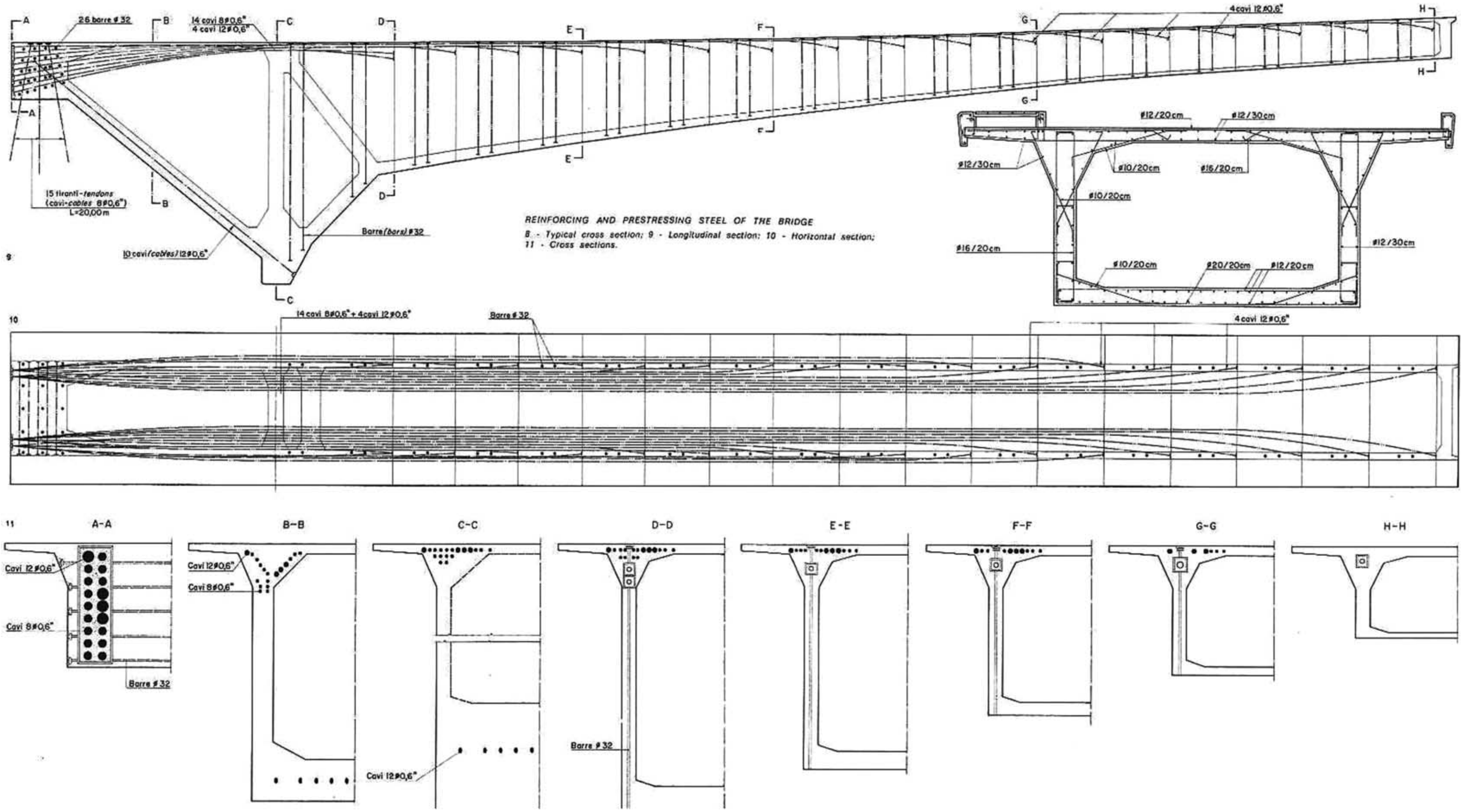


Fig. 4. Layout of the prestressing cables and some details about the distribution of the main reinforcement bars (reprinted with permission from *L'industria Italiana del Cemento*—[7]).

$$f_{c,\mathrm{nom}} = -31.8\ \mathrm{MPa},\ E_c = 30\ \mathrm{GPa},\quad \varepsilon_{cu} = -2.5‰, \quad (10a)$$

$$f_{sy,\mathrm{nom}} = 500\ \mathrm{MPa},\ E_s = 210\ \mathrm{GPa},\ \varepsilon_{su} = 1\%, \quad (10b)$$

$$f_{py,\mathrm{nom}} = 1940\ \mathrm{MPa}, E_p = 200\ \mathrm{GPa},\ \varepsilon_{pu} = 1\% \quad (10c)$$

with a nominal weight density $\gamma_{\mathrm{nom}} = 20\ \mathrm{kN/m^3}$.

The analysis is aimed to investigate the reliability of the bridge with respect to a change of the traffic load category.

5.1. Structural model

The constitutive laws adopted for materials are shown in Fig. 5 [4]. The stress–strain diagram of concrete in compression is described by Saenz (Fig. 5a), with initial modulus $E_{c0} = 9500 f_c^{1/3}$ and peak strain $\varepsilon_{c1} = -2‰$. In tension concrete is assumed elastic perfectly plastic, with tensile strength $f_{ct} = 0.25 f_c^{2/3}$ and ultimate tensile strain $\varepsilon_{ctu} = 2 f_{ct}/E_{c0}$. The stress–strain diagram of reinforcing steel is assumed elastic perfectly plastic both in tension and in compression (Fig. 5b). For prestressing steel the plastic branch is assumed non-linear and described by a fifth order degree polynomial function (Fig. 5c).

The bridge structure is modeled by using a prestressed concrete beam finite element whose formulation, based on the Bernoulli–Navier hypothesis, deals with both material and geometrical non-linearity [3,5]. Fig. 6 shows the two-dimensional framed model, while the corresponding modeling of some typical cross-sections are shown in Fig. 7. Fig. 6 also shows the results obtained through a non-linear analysis under a uniform distributed load. With regards to the accuracy of such results, no measured data is available for the structure examined. However, the procedure of static non-linear analysis of two-dimensional framed structures used in this application has been widely tested on a series of benchmarks presented in Bontempi et al. [3] and Malerba [5]. Additional benchmarks dealing with three-dimensional framed structures under cyclic static and dynamic excitations can be found in Biondini [2].

5.2. Random variables

The basic random variables **X** used in the simulation are listed in Table 1 [8]. In the following, the probabilistic models are briefly described. Unless correlation is explicitly specified, statistical independence between random variables is assumed.

For material models, the parameters ε_{ctu}, ε_{c1}, ε_{cu}, ε_{su}, ε_{pu}, E_s, E_p, are assumed deterministic, while f_c, f_{sy}, f_{py}, are considered lognormally distributed random variables with mean values equal to the nominal ones and standard deviations of 5, 10 and 100 MPa, respectively.

The geometrical parameters considered as random variables are (a) the location (x, y) of the nodes of the structural elements; (b) the linear dimensions d of the boundaries of their cross-sections; (c) the depths y_s and y_p, and (d) the areas A_{s1} and A_{p1}, of each reinforcing and prestressing bar, respectively. These variables are taken as normally distributed with mean values equal to the nominal ones and standard deviations of 50, 5, 5 mm, and $0.025 A_{1,\mathrm{nom}}$, respectively (see Table 1).

The prestressing force P is taken as a random variable uniformly distributed between the values $\lambda_{\min} P_{\mathrm{nom}}$ and $\lambda_{\max} P_{\mathrm{nom}}$. Due to the high uncertainty in the prestressing force, the values $\lambda_{\min} = 0$ and $\lambda_{\max} = 1$ are assumed.

The dead load G, including the weight of structural and non-structural members, is considered as a normally distributed random variable, with mean value equal to the nominal one and with a coefficient of variation of 10%. The live loads are derived for each lane by a suitable combination of the following uniform loads:

(a) module of length 10.50 m and intensity 55 kN/m (heavy vehicle);

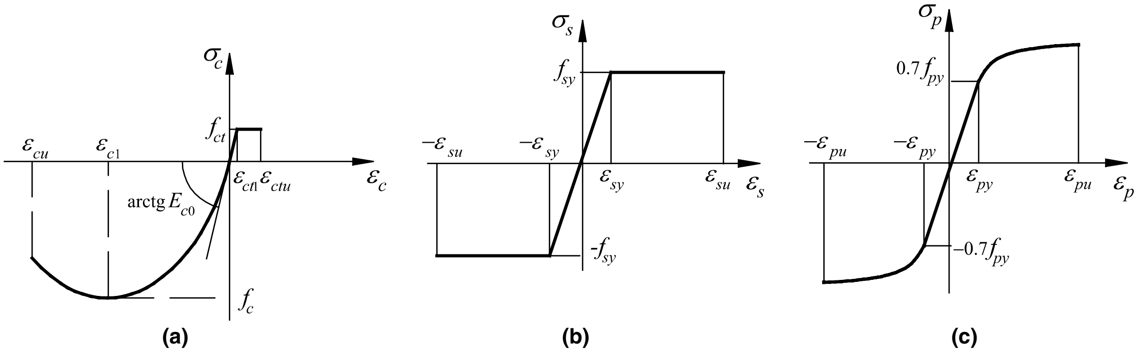


Fig. 5. Stress–strain diagrams of the materials: (a) concrete, (b) reinforcing steel and (c) prestressing steel.

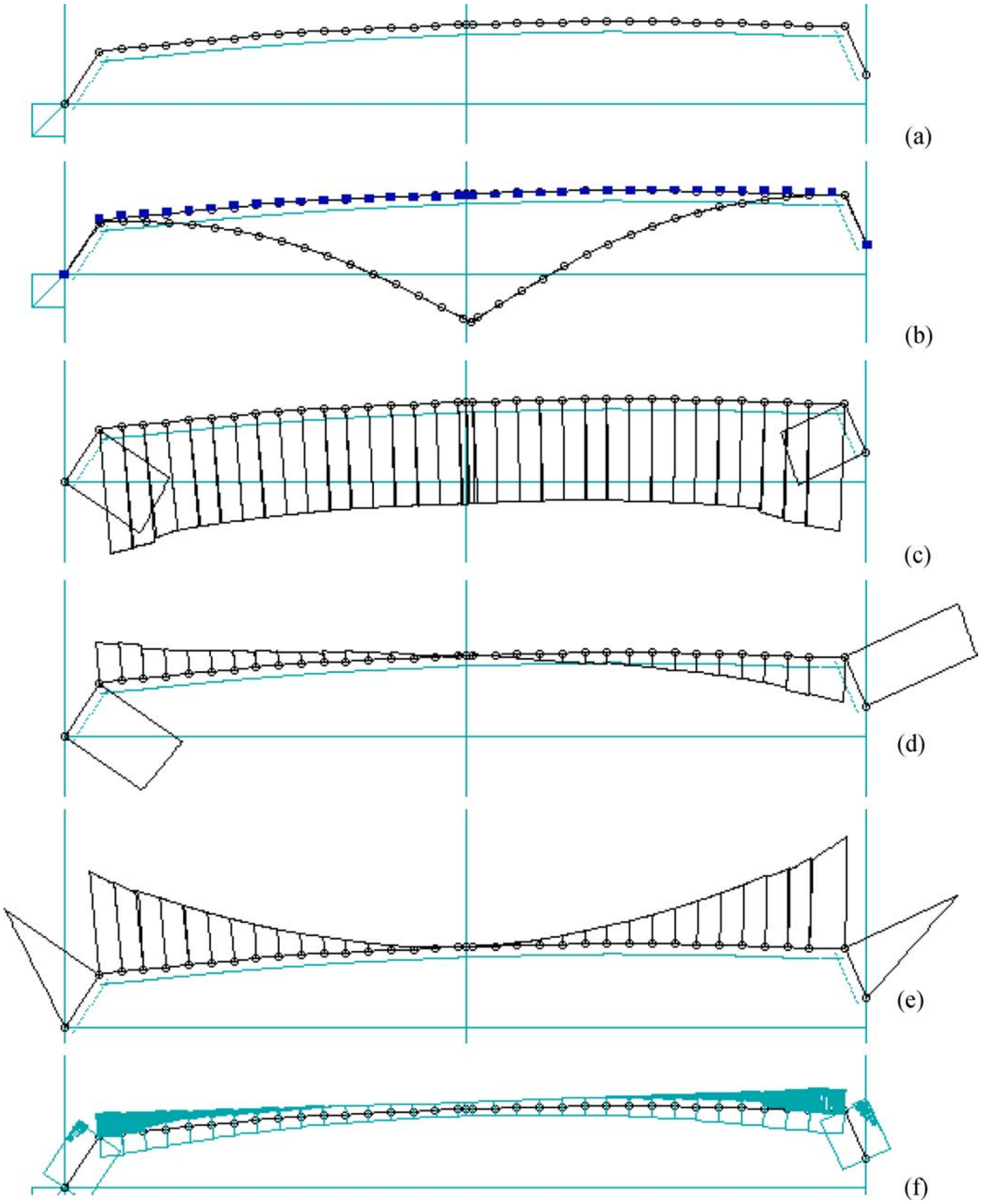


Fig. 6. Model of the bridge and results of the structural analysis at collapse for a uniform distributed live load: (a) framed model; (b) deformed shape; (c) axial force; (d) shear; (e) bending moment; (f) cracking pattern (shaded).

(b) module of variable length 10.50 m and intensity 15 kN/m (normal traffic).

In particular, the following load combinations are considered:

Case 1: a number of loads type (a) with the possible presence of a load type (b) on both lanes;

Case 2: one load type (a) with the possible presence of a load type (b) on both lanes;

Case 3: one load type (a) with the possible presence of a load type (b) only on one lane, while on the other lane only a load type (b) is present;

Case 4: one load type (a).

Fig. 8 shows some typical live load distributions for the load combinations associated with cases 1 and 3.

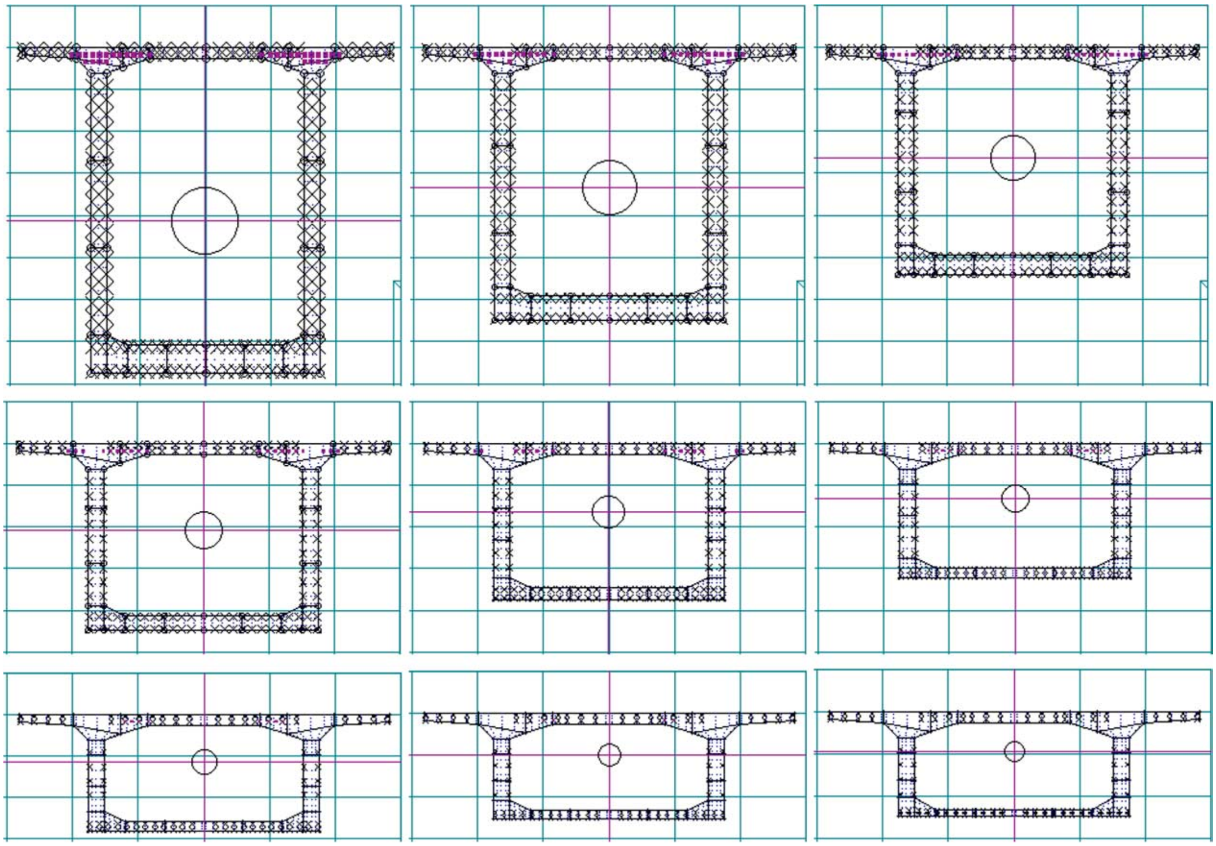


Fig. 7. Modeling of the composite cross-sections: (•) prestressing cables; (×) reinforcement bars.

Table 1
Probability distributions and their parameters (mean value μ and standard deviation σ)

Random variables	Distribution type	μ	σ
Concrete strength, f_c	Lognormal	$f_{c,\mathrm{nom}}$	5 MPa
Reinforcing steel strength, f_{sy}	Lognormal	$f_{sy,\mathrm{nom}}$	30 MPa
Prestressing steel strength, f_{py}	Lognormal	$f_{py,\mathrm{nom}}$	100 MPa
Coordinates of the nodal points (x, y)	Normal	$(x, y)_{\mathrm{nom}}$	50 mm
Cross-sectional dimensions, d	Normal	d_{nom}	5 mm
Depth of steel bars and cables, y	Normal	y_{nom}	5 mm
Area of steel bars and cables, A_1	Normal	$A_{1\mathrm{nom}}$	$0.025A_{1\mathrm{nom}}$
Dead loads, G	Normal	G_{nom}	$0.10G_{\mathrm{nom}}$
Live loads, Q	Normal	Q_{nom}	$0.40G_{\mathrm{nom}}$

Due to lack of information, prestressing forces P are assumed to be uniformly distributed between $P = 0$ and $P = P_{\mathrm{nom}}$.

5.3. Numerical simulation and reliability assessment

The serviceability limit states are detected by assuming $\alpha_c = 0.45$, $\alpha_s = 0.60$, $\alpha_p = 1.10$, $s^+ = -s^- = l_{\mathrm{nom}}/400$, with $l_{\mathrm{nom}} = 125$ m. Fig. 9 shows the cumulative distributions $F_\Theta(\theta)$ of the probabilistic safety factors with the consideration of serviceability and ultimate limit states for different load conditions. It is worth noting that the limit states S1 and S3 are not shown since they are associated with very large, $P_F \approx 1$, and very small, $P_F \approx 0$, probabilities of failure, respectively. In particular, Fig. 9 shows a comparison between the cumulative functions $F_\Theta(\theta)$ given by samples of 1000 simulations with those derived from the regression of the data, as well as the corresponding reliability indices $\beta = -\Phi^{-1}[F_\Theta(1)]$. The number of simulations has been properly chosen in such a way that a relatively stable mean and standard deviation values of the reliability indices are obtained.

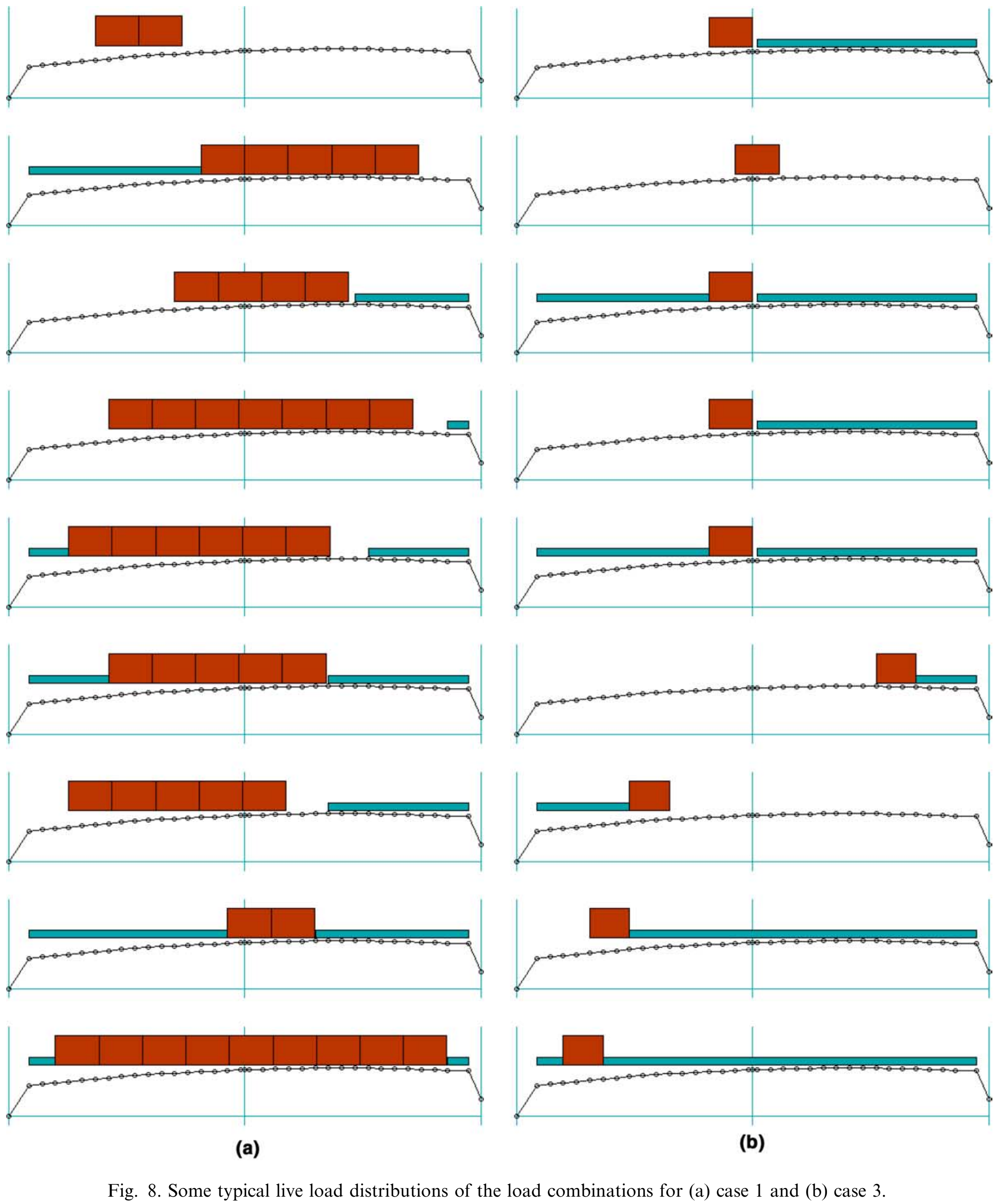
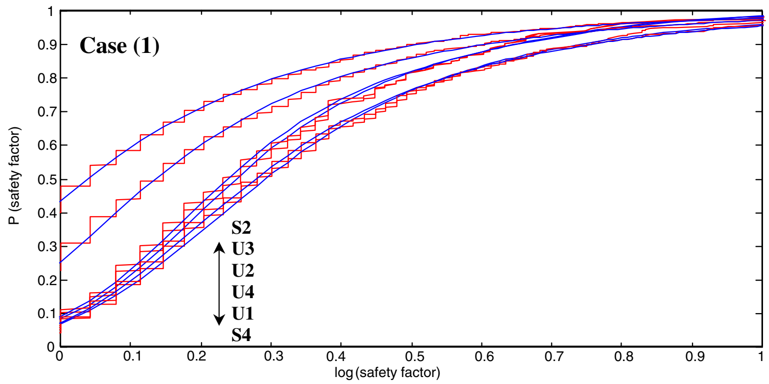


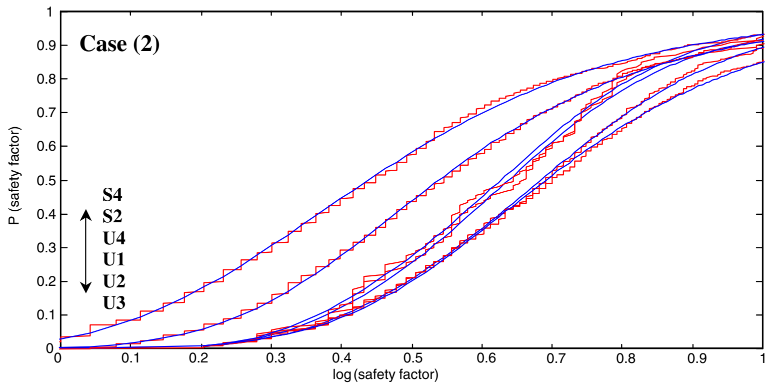
Fig. 8. Some typical live load distributions of the load combinations for (a) case 1 and (b) case 3.

6. Conclusions

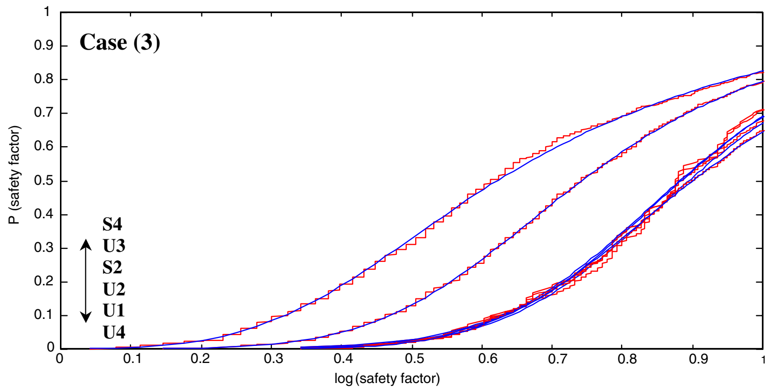
A direct and systematic approach to the reliability analysis of reinforced and prestressed concrete structures subjected to static loads has been presented. The effectiveness of the Monte Carlo Method in assessing the reliability of this class of structures has been investigated and the fundamental role played by a robust non-linear structural analysis leading to a full exploration of all the serviceability and ultimate limit states is emphasized. Special attention has been devoted to the reliability analysis of existing structures and an arch bridge has



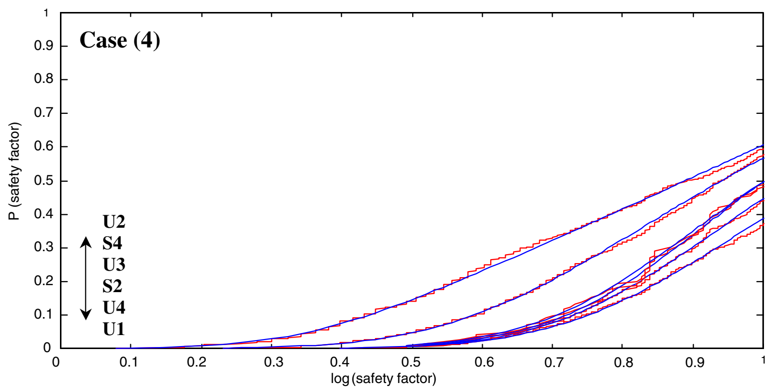
Limit State	β-index
S2	0.638
S4	1.440
U1	1.420
U2	1.380
U3	1.335
U4	1.404



Limit State	β-index
S2	2.591
S4	1.845
U1	4.009
U2	4.627
U3	4.640
U4	3.431



Limit State	β-index
S2	3.731
S4	3.171
U1	4.224
U2	3.790
U3	3.367
U4	5.072



Limit State	β-index
S2	4.268
S4	3.760
U1	5.990
U2	3.715
U3	4.012
U4	5.315

Fig. 9. Cumulative distribution $F_\Theta(\theta) = P$ (safety factor) versus $\log\theta = \log$(safety factor), and reliability indices $\beta = -\Phi^{-1}[F_\Theta(1)]$ for both serviceability and ultimate limit states for different load conditions.

been selected as structural prototype in order to verify the effectiveness of the proposed approach. Such bridge represents a real "case study" where reliability analysis has been actually selected as the main tool for the evaluation of the structural performance of an existing structure under loads sensibly higher than the original design loads (change of the traffic load category).

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